CVE 307 HANDOUT NOTES

1st lecture · CVE 307   12/01/2018 Friday   (2-4) Tuesdays.

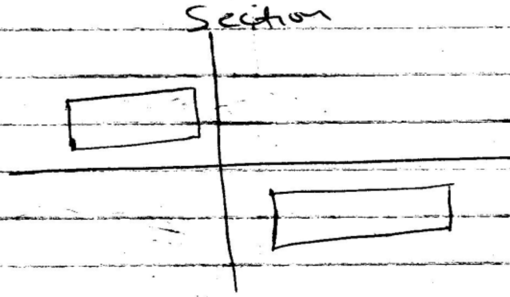
ADVANCE TOPIC IN BENDING MOMENT AND SHEAR FORCE IN BEAMS.   Pg1

What is Shear force? Algebraic sum of the vertical forces at any section of a beam to the right or left of the section

What is BM? Algebraic sum of ~~the~~ the moments of all the forces acting to the right or left of the section.

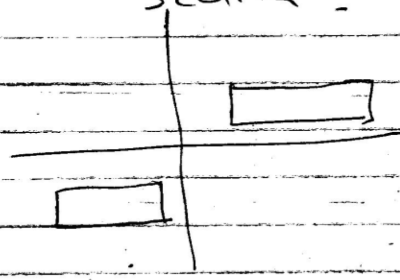
~~Sung~~ SIGNS CONNECTIONS

Section



Positive Shear force

Section



Negative Shear forc

Concavity a Top

Sagging

Positive B.M

Convexity at Top

Hogging

Negative B.M.

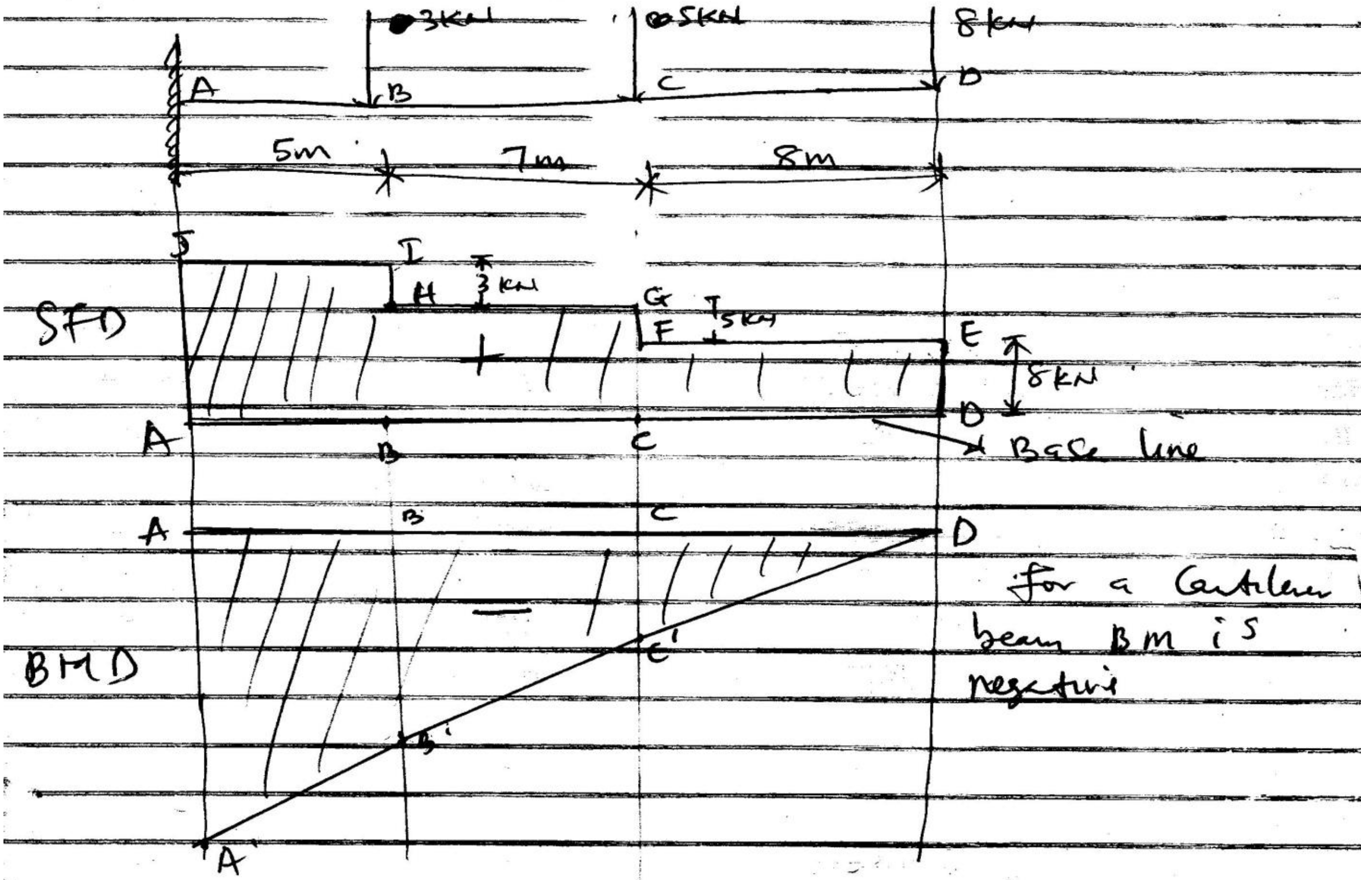
CVE 307   Pg 1
2019/2020

Tue. 11/02/2020

CVE 307 Lecture
10

SHEAR FORCE & BM FOR A CANTILEVER LOAD AT THE WITH POINT

Problem 1. A cantilever beam of length 2m carries the point load as shown below. Draw the shear force and B.M diagrams for the cantilever beam.



Shear force Diagram.

SF at D = +8KN. This S.F remains constant between D and C. At C, due to point load, the shear force becomes (8+5) = 13KN. Between C and B, the shear force remains 13KN. At B again, S.F becomes 13+3 = 16KN. S.F between B and A remains constant and equal to 16KN. Hence

SF at D = F_D = +8 KN

SF at C = F_C = +8 + 5 = +13 KN

SF at B = F_B = +13KN + 3 = +16 KN

SF at A = F_A = +16 KN −

20/4/2020 Pg 2 11/02/2020

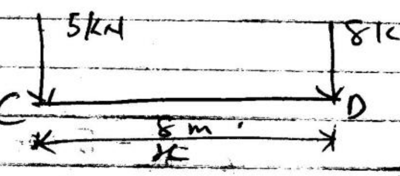
FREE END

CVG 307 17/18 1st lecture.

Pg2

SF DIAGRAM.

Draw a horizontal line AD as base line. On the base ~~line~~ (AD) mark the points B and C below the point loads. Take ordinate DE = 8kN in the upward direction. Draw EF parallel to AD. The point F is vertically above C. Take vertical line FG = 5kN. Through G, draw a horizontal line GH in which point H is vertically above B. Draw vertical line HI = 3kN. From I draw a horizontal line. The point J is vertically above A. This completes the shear force diagram.

(i) 

Bending moment.

BM at D : $M_D = 0$

BM at any section between C & D at a distance x from D is given by

$M_x = -8 \times x$ which follows a straight line law

At C : $x = 8m$ $M_C = -8 \times 18 = -64$ kNm.

(ii) BM at any section b/w B and C at a distance x from D is given by

~~$M_x = -8x - 5(x-8) - 3(x$~~

~~At~~ x = 8m and at B, x = 8+7m = 15m.

Hence here x varies from 8 to 15

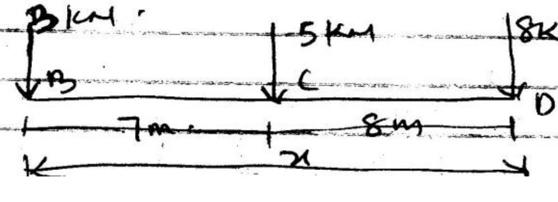
$M_x = -8x - 5(x-8)$

BM between B and C also varies by a straight line law

BM at B is obtained by substituting x = 15m.

i.e. $M_B = +8 \times 15 - 5(15-8)$.

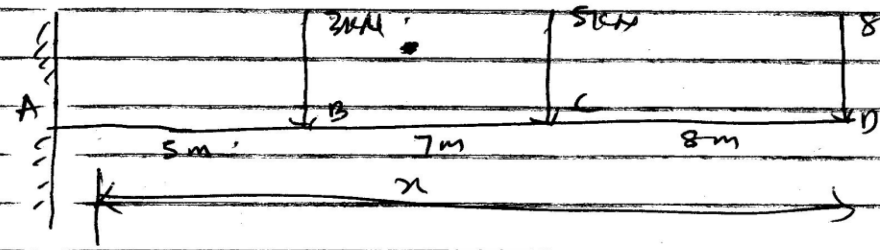
$= 120 - 35 = -155$ kNm.

(ii) 

at x = 15m.

2019 2020 Pg3

(iii) B.M at any section between A and B at a distance x from D is given by



$$M_x = -8x - 5(x-8) - 3(x-15)$$

BM b/w A and B varies by a straight line law.

BM at A is obtained by substituting $x = 20m$

i.e $M_A = -8 \times 20 - 5(20-8) - 3(20-15)$
$= -160 - 60 - 15 = -235 \text{ kN/m}$.

BM diagram:

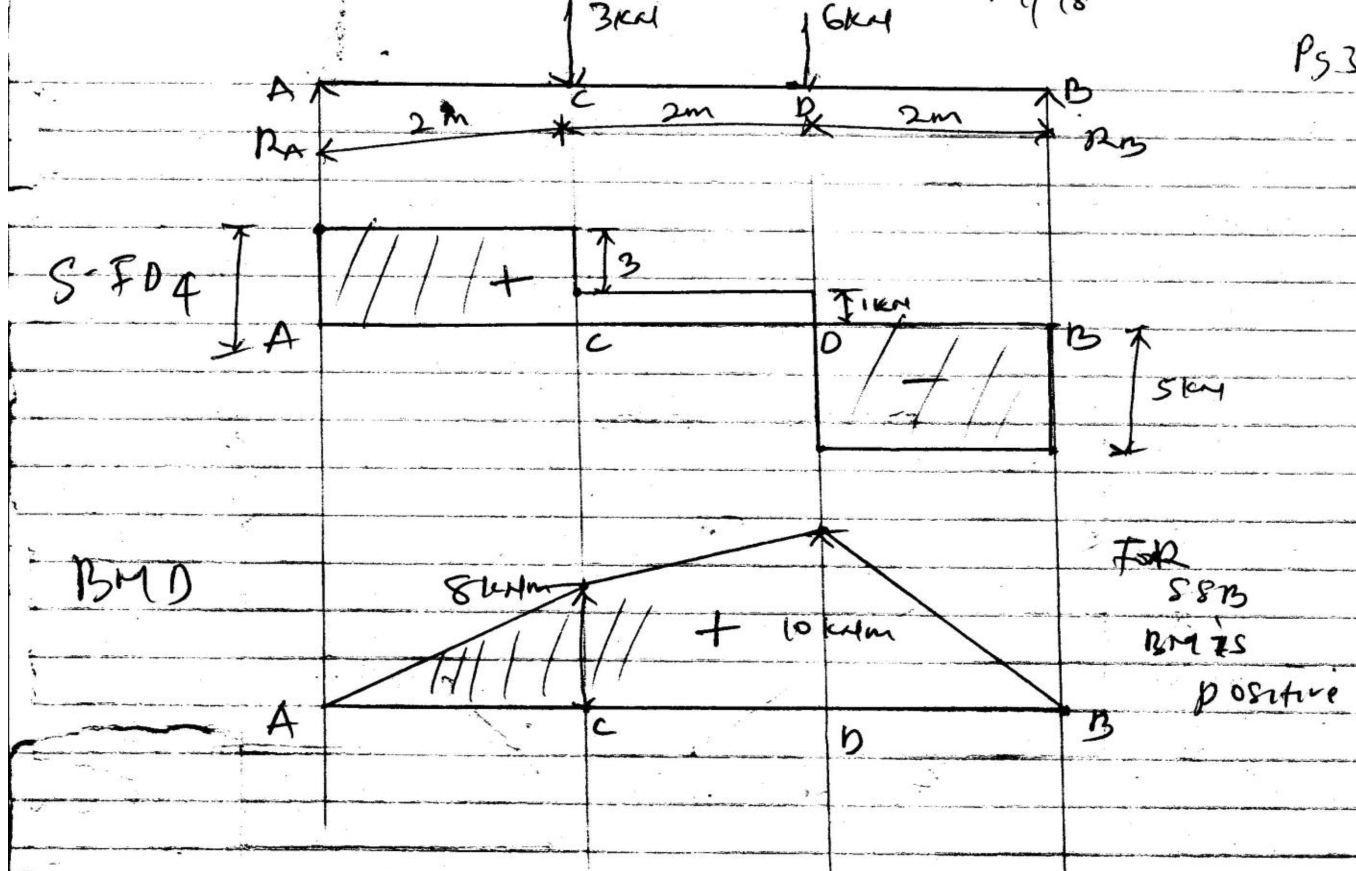
Draw a horizontal line AD as a base line and mark the points B and C on this line. Take vertical lines $CC' = 64$ kN/m, $BB' = 155$ kN/m and $AA' = 235$ kN/m in the downward direction. Join D, C', B', A by a straight lines. This completes the BM diagram

(2) SF & BM DIAGRAM FOR A SSB WITH AN ECCENTRIC POINT LOAD.

A SSB of length 6m carries point load of 3kN and 6kN at distances of 2m and 4m from left end. Draw S.F and B.M diagrams for the beam.

PS 4 11/02/2020

CVE307 17/18



Reactions

$\Sigma M_A = 0$

$R_B \times 6 = 3\times2 + 6\times4$

$6R_B = 30$

$R_B = \underline{\underline{5kN}}$

$R_A = 9 - 5 = \underline{\underline{4kN}}$

SF Diagram

At A, $F_A = +4kN$

At C $F_C = +4 - 3 = +1\,kN$

At D $F_D = +1 - 6 = -5kN$

At B $F_B = -5\,kN$.

BM

At A, $M_A = 0$

At C $M_C = R_A \times 2 = +8kNm$

At D $M_D = R_A \times 4 - 3\times2$

$= 8\times4 - 3\times2 = +10kNm$

$M_B = 0$ or

$M_B = R_A \times 6 - 3\times4 - 6\times2$

$= 24 - 12 - 12 = 0$

proved

Shear F. b/w A & C is constant $= +4kN$

SF b/w C & D is constant $= +1kN$

SF b/w D & B is constant $= -5kN$.

SF at B $= F_B = -5kN$,

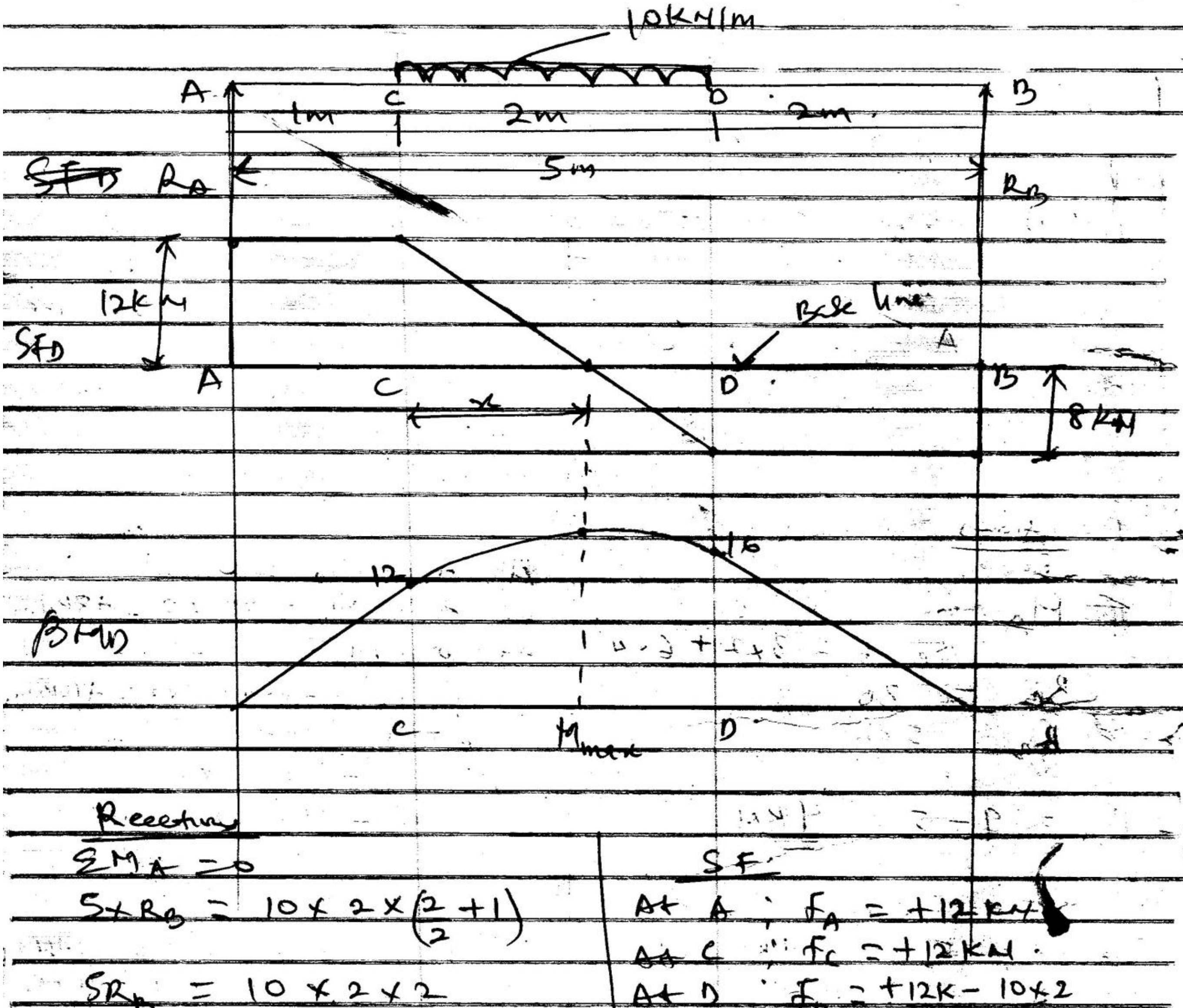
11/02/2020 Pg 5

307 18 lecture · 12/18

(3)

SF & BM · FOR A UDL

A Simply Supported beam 5m long with a UDL of 10kN/m over a length of 2m as shown below.
Draw SF & BM diagrams for the beam indicating the value of maximum bending moment B_{max}.



Reaction

$\Sigma M_A = 0$

$5 \times R_B = 10 \times 2 \times \left(\frac{2}{2} + 1\right)$

$5R_B = 10 \times 2 \times 2$

$5R_B = 40$

$R_B = 8kN$

$R_A = (10 \times 2) - 8$

$R_A = 12kN.$

SF

At A ; $F_A = +12kN$

At C ; $F_C = +12kN.$

At D ; $F_D = +12k - 10 \times 2$

$= -8kN$

At B ; $F_B = -8kN$

p6 6 11/02/2020

19/3/19 done

BM

At A, $M_A = 0$

At C, $M_C = R_A \times 1 = 12$ KNm

At D $M_D = R_A \times 3 - 10 \times 2 \times 1$

$= 36 - 20 = 16$ KNm

At B, $M_B = 0$

OR

$M_B = R_A \times 5 - 10 \times 2 \times \left[\frac{2}{2} + 2\right]$

$= 60 - (20 \times 3) = 0.$

proved.

To find M_{max}

M_{max} occurs at a point where SF changes sign i.e where SF is zero. From SFD let x represent the distance between C and M_{max}. From the geometry of the figure between C and D

SF btw A & C constant

Still constant $= +12$KN

SF b/w C & D changes to -8KN at D.

Shear f b/w D & B constant.

$= -8$KN.

11/02/2020 PT

CVE307

Pg 9

We find that

$\frac{x}{12} = \frac{2-x}{8}$

$8x = 24 - 12x$

$20x = 24$

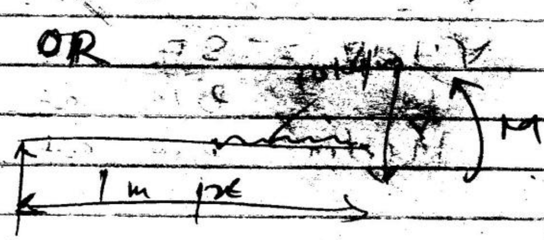
$x = 1.2$m.

$M_{max} = R_A(1+x) - 10 \times x \times \frac{x}{2}$

i.e $= 12(1+1.2) - 10 \times 1.2 \times \frac{1.2}{2}$

$= 19.2$ KNm.

OR



$R_A = 10x + V$

$12 = 10x + V$

$V = 12 - 10x$

at $V = 0$

$-10x = -12$

$x = 1.2$ m.

$R_A(1+x) - \frac{10x^2}{2} = M_x$

at $x = 1.2$ M_{max}

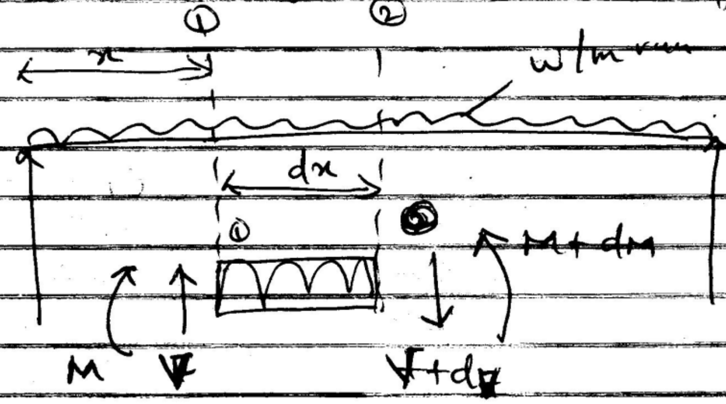
$= 19.2$ KNm.

2019/2020 — Lecture 1 ends

✓ 19/3/19 done

Relations Btw load, Shear Force and Bending moment.

Figure below showed a beam carrying a UDL of w per unit length. Consider the equilibrium of the portion of the beam $\boxed{\text{btw}}$ Sections 1 – 1 and 2 – 2. This portion is at a distance x from left supports and is of length dx.



V = Shear F at Section 1 – 1

$V + dV$ = SF at Section 2 – 2

M = BM at Section 1 – 1

$M + dM$ = BM at Section 2 – 2

There4, the forces and moments acting on the length 'dx' of the beam are.

Equilibrium of forces in vertical direction is given as

$$V - w\,dx - (V + dV) = 0$$

$$V - w\,dx - \cancel{V} - dV = 0$$

$$-dV = w\,dx$$

$$\frac{dV}{dx} = -w$$

above ~~def~~ equation shows that the rate of change

Pg 5

(V)
of shear force is equal to the rate of loading (w)

Integrating

i.e $\int_A^B dV = \int_A^B -w\,dx$

i.e $V_B - V_A = -\int_A^B w\,dx$

Area of loading diagram between A & B

The area of loading diagram may be +ve or negative.

Moment equilibrium of element

$M - w\,dx \cdot \frac{dx}{2} + V\,dx = M + dM$

$M - \frac{w\,dx^2}{2} + V\,dx = M + dM$

$dM = -\frac{w\,dx^2}{2} + V \cdot dx$

dx is infinitesimally small therefore the higher powers will tend to zero!

i.e $dM = V\,d \cdot x$

$\frac{dM}{dx} = V \Rightarrow V = \frac{dM}{dx}$

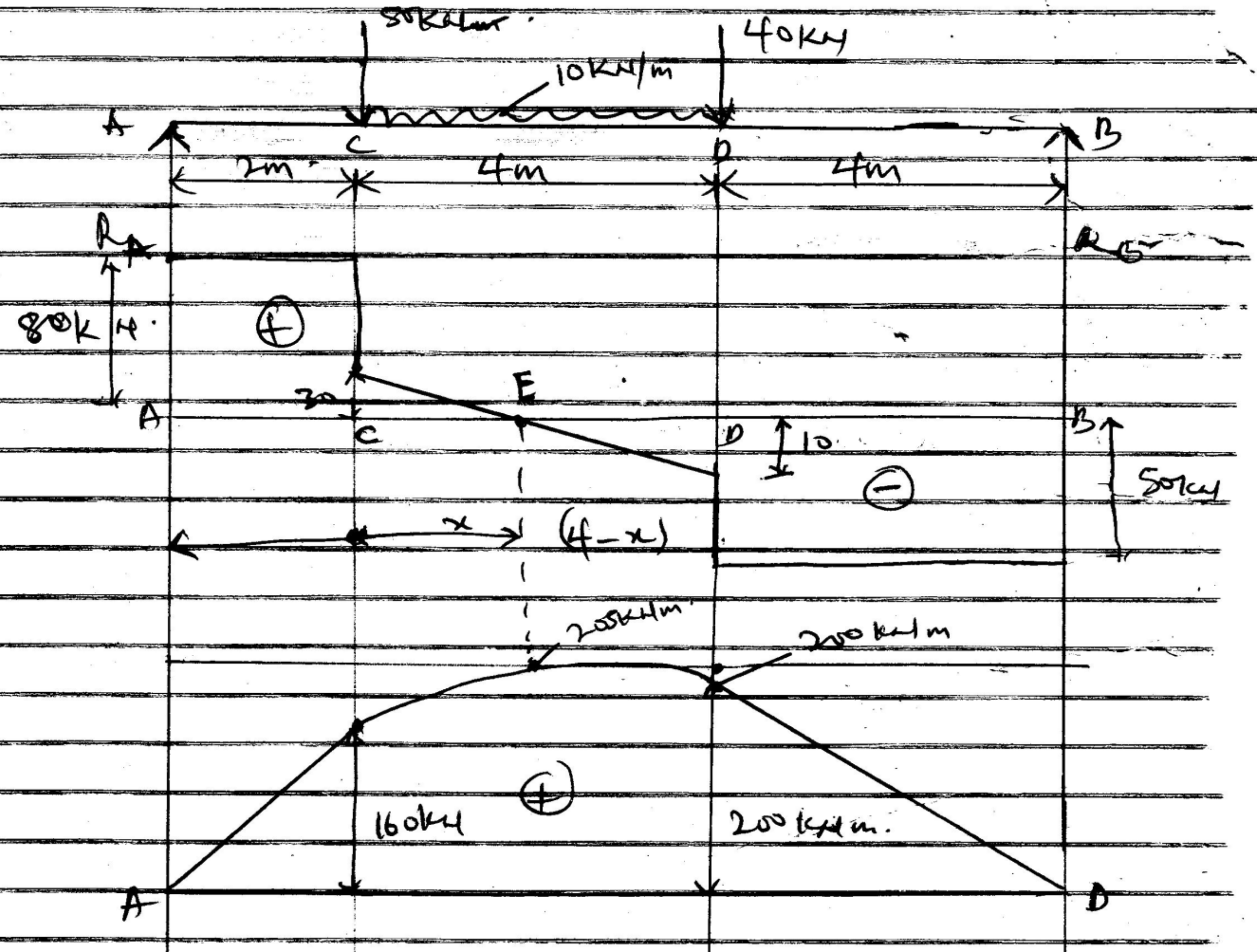
Maximum or Minimum BM occurs at $\frac{dM}{dx} = 0$ i.e at $V = 0$

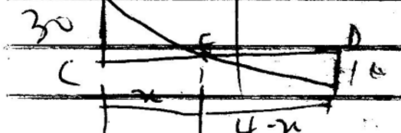
19/3/19

The above equation shows that the rate of change of BM is equal to the SF of the section.

Problem

Q. SSB of length 10m carries UDL ~~load~~ and two point loads as shown below. Draw S.F and B.M diagrams for the beam. Also calculate the maximum bending moment.





$\frac{x}{30} = \frac{4-x}{10}$ $x = 3m$

$M = R_A(2+x) - 30x - 10x \times \frac{x}{2}$

Pg 36

Reactions

$\Sigma M_B = 0$

$10R_A = 50\times 8 + 40\times 4 + 10\times 4\times 6$

$= 400 + 160 + 240$

$10R_A = 800$

$R_A = 80$ kN

$R_A + R_B = 50 + 40 + 40$

$R_B = 130 - 80$

$= 50$ kN

SF

$V_A = +80$ kN

$V_C = +80 - 50 = +30$ kN

$V_D = +30 - 10\times 4 - 40 = -50$ kN

$V_B = -50$ kN

BM

$M_A = 0$

$M_C = 80\times 2 = 160$ kNm

$M_D = 80\times 6 - 50\times 4 - 10\times 4\times 2$

$= 480 - 200 - 80$

$M_D = 200$ kNm

$M_B = 0$

M_{max} it occurs at where $V = 0$

$\boxed{\text{Geometry } \frac{x}{30} = \frac{4-x}{40}}$

$\boxed{\begin{aligned} 10x &= 120 - 30x \\ 40x &= 120 \\ x &= \frac{120}{40} = 3m \end{aligned}}$

let distance E from A $= x$

Shear force at E $= V_E$

$V_E = R_A - 50 - 10(x-2)$

$= 80 - 50 - 10x + 20$

$V_E = 50 - 10x$

at $V_E = 0$; M_{max} occurs

i.e.

$50 - 10x = 0$

$x = 5$ m

$\cancel{M_E = R_A(2+x) - 50x - \frac{10x^2}{2}}$

$\cancel{M_{max} \text{ at } x = 5m}$

$\cancel{M_{max} = 80\times 7 - 50\times 5 - 5\times 5^2}$

$\cancel{= 560 - 250 - 125}$

$M_E = R_A x - 50(x-2) - 10(x-2)\frac{(x-2)}{2}$

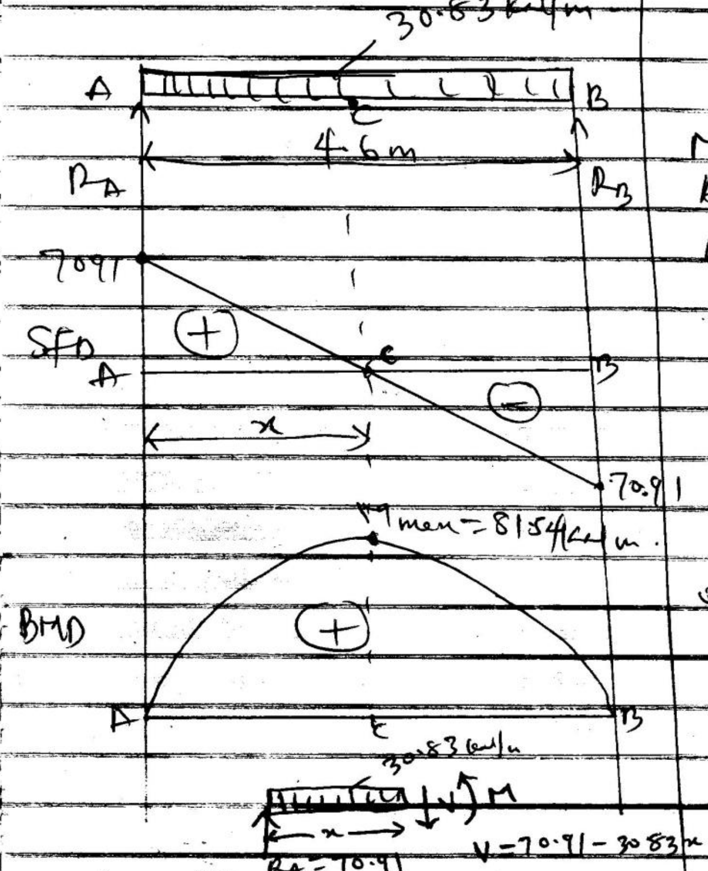
$= 80\times 5 - 50\times 3 - 30\times\frac{3}{2}$

$= 400 - 150 - 45$

$= 205$ kNm

CVE 317

23/01/18 Monday



Reactions

$\circlearrowleft M_B = 0$

$4.6R_A - 30.83 \times 4.6 \times \frac{4.6}{2} = 0$

$4.6R_A = 326.18$

$R_A = 70.91$ kN

$R_A + R_B = 30.83 + 4.6$

$R_B = 141.818 - 70.91$

$R_B = 70.91$ kN.

SF

At A, $V_A = 70.91$

At C, $V_C = 70.91 - 30.83 \times 2.3$

~~= 0 − 70.91 = 70.91~~

$V_C = 0$

① beginning

At B, $V_B = 0 - 70.91$

$= -70.91$ kN

Bending moment

$M = R_A x - \frac{30.83x^2}{2}$

At A, $M_A = 0$

At C, $M_C = 70.91 \times 2.3 - 30.83 \times 2.3 \times \frac{2.3}{2}$

$= 163.093 - 81.54$

$M_C = 81.54$ kN/m.

$M_B = 0$

or

$M_B =$ [illegible] $\times 4.6 \times \frac{4.6}{2}$

$= 326.186 - 326.1814$

$= 0$

M_{max} occurs at a point where $V = 0$ (C)

let distance from A be $x =$

We should know that $x = 2.3$ (Viewing)

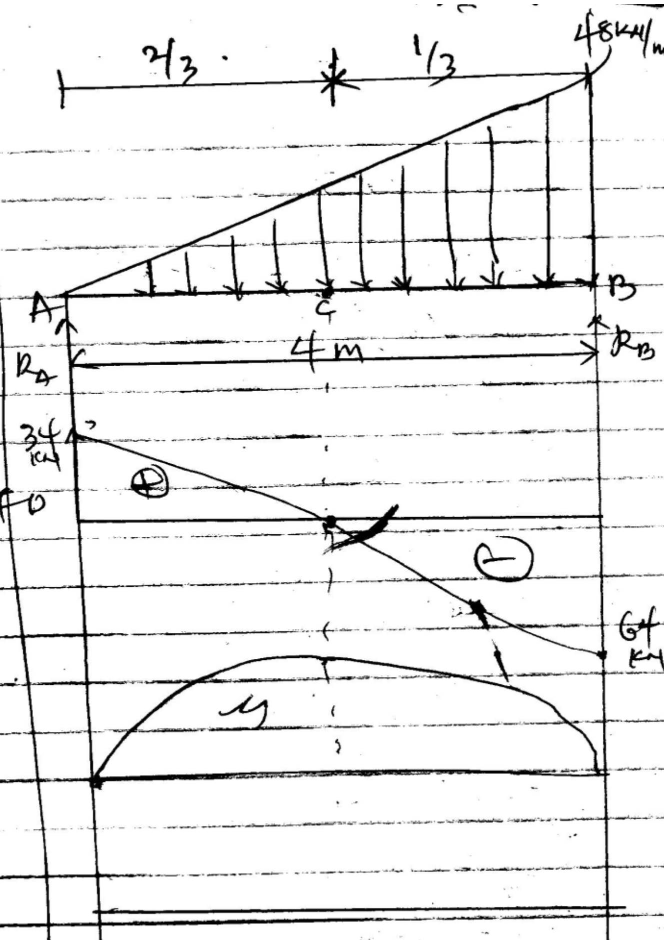
But

$V_C = R_A - 30.83x$

~~$R_A x - \frac{30.83x^2}{2} = V_C$~~

or $R_A - 30.83x - V_C = 0$

PS 7 (2.7) ① SOA



~~$70.91x - 30.8\frac{x^2}{2} = V_c$~~

$70.91 - 30.8x = V_c$

at $V_c = 0$

$30.8x = 70.91$

$x = 2.3$ m.

$M_{max} = M_c$

$R_A x - 30.83\, x \cdot \frac{x}{2} = M_{max}$

$70.91x - 30.83\frac{x^2}{2} = M_{max}$

at $x = 2.3$

$163.093 - 81.54$

$M_{max} = 81.54$ kN m.

Reaction. $\Sigma M_B = 0$

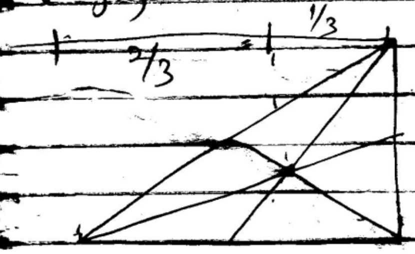
$4R_A - \left[\frac{1}{2} \times 48 \times 4\right] \times \frac{1}{3} \times 4 = 0$

$R_A = 128/4 = 32$ kN

$R_B = \left[\frac{1}{2} \times 48 \times 4\right] - 32$

$= 64$ kN

② Simply supported with Gradually Varying load



Total load = Area of Triangle

$\frac{1}{2} B \times H$

Centroid or Centre of Gravity CG (Where) Centroid and CG are as the same point.

CG of a body is the point through which the whole weight of the body acts.

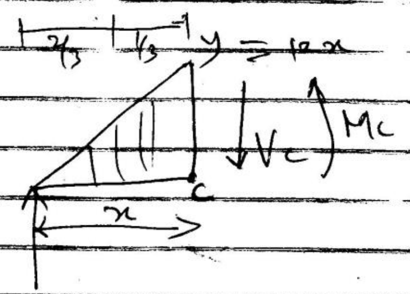
Centroid : The point at which the total area of a plane figure (rectangle, square, triangle, quadrilateral, circle etc) is assumed to be concentrated is called the centroid of that area.

$y = x^n$; $dy/dx = nx^{n-1}$

Shear force.

~~At~~ A : $V_A = 32$ kN

At C : $V_C = 32 - 6x^2$



$R_A = 32$ kN.

Similar Δ

$\frac{y}{x} = \frac{48}{4}$

$\frac{y}{x} = 12$

$y = 12x$

$V_c = R_A - \frac{1}{2} \times (y \times x)$

$= R_A - \frac{1}{2} \times 12x \times x$

$V_c = R_A - 6x^2$

At C = Shear force is equal to zero (Noted)

i.e $R_A - 6x^2 = 0$

$6x^2 = 32$

$x^2 = 5.3$

$x = 2.309$ m.

$V_c = 32 - 6(2.309)^2$

$V_c = 0$

(2) $y = ax^n$: $\frac{dy}{dx} = nx^{n-1}$

$V_B = 0 - 64$

$= -64$ kN.

BM

At A, $M_A = 0$ (x)

At C $M_C = R_A \times 2.309 - \frac{yx}{2} \times \frac{1x}{3}$

$M_C = R_A x - \frac{\cancel{12}^{2} x^2 \cdot x}{\cancel{6}}$

$M_c = R_A x - 2x^3$

at $x = 2.309$, $M_c = M_{max}$

$M_{max} = 32 \times 2.309 - 2(2.309)^3$

$= 73.888 - \cancel{24.621}$

$= 49.27$ kNm

Give Student Assignment

$M = \int V$

$V = R_A - \frac{yx}{2}$

$y = 12x$

$V = R_A - 6x^2$

$M = \int V$

$M = R_a x - 2x^3$

$V = \frac{dM}{dx}$

$V = R_a - 6x^2$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$ $(n \neq -1)$

Summary

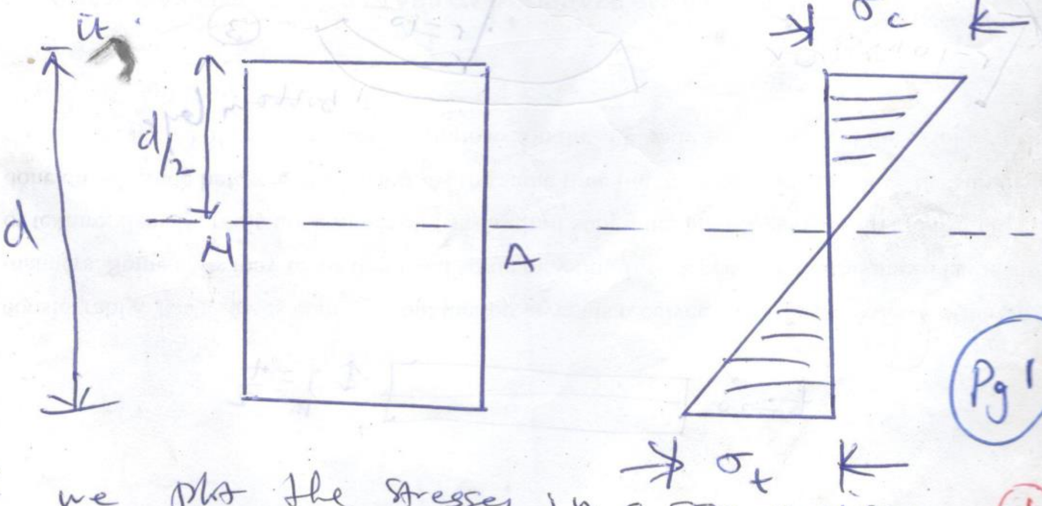
CVE 307

CVE 307

BENDING STRESS IN SYMMETRICAL SECTIONS

The neutral axis (N·A) of a Symmetrical Sections such as circular, rectangular or Square) lies at a distance $d/2$ from the outermost layer of the section where d is the diameter (for circular section) or depth for a rectangular or a square sections.

- There is no stress at the N·A. but stress at a point is directly proportional to its distance from the N·A.
- The maximum stress takes place at the outermost layer.
- For a SSB, there is a compressive stress above the N-A. and tensile stress below it.



If we plot the stresses in a SSB section, we shall get the figure above

Pg 1

13

CVE 307

~~Ben~~

Bending stress in Unsymmetrical Sections

In case of symmetrical sections, ~~the geometrical centre of the~~ the N.A passes through the geometrical centre of the sections. But in case of unsymmetrical sections such as L, T sections, the neutral axis do not pass through the geometrical centre of the sections.

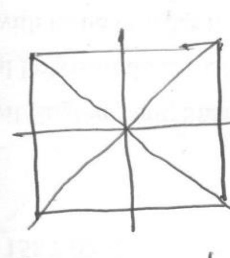
Hence the value of y (distance from the N.A to the outermost part) from the topmost layer or bottom layer of the section from the N.A wont be the same.

As we know, for finding the bending stress in the beam, the bigger ⟹ ⟹

Something is [Excluded]
Symmetrical has corresponding similar parts. In other words, one side is the same as the other.

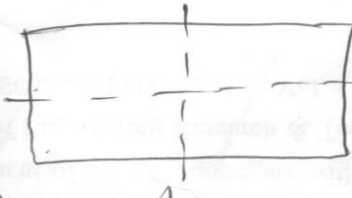
- If you can draw a line down the center of something and get two similar halves, it is Symmetrical

Lines of Symmetry



[Excluded]

Square has 4 lines of symmetry.



Rectangle
2 lines of symmetry.

P

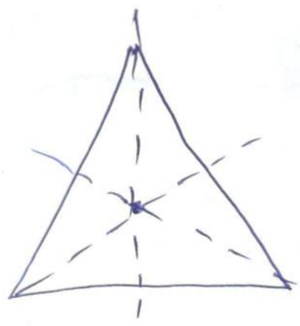
(Pg 2)

Value of y is used.
As N.A passes through the Centre of gravity of the section, hence, in unsymmetrical section, first the C.G is calculated first.

C.G

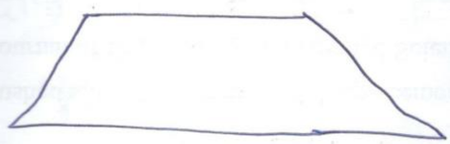
Centre of gravity of a body is the point through which the whole weight of the body acts.
A body is having only one C.G for all positions of the body
It is represented by C.G or simply G.

Pg 3

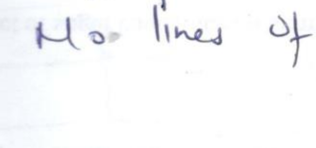


Equilateral triangle.
3 lines of symmetry.

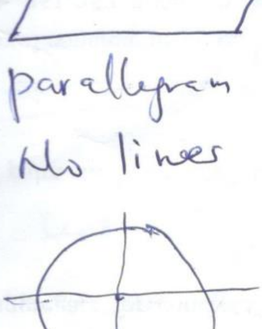
Isosceles triangle
1 line of symmetry.



Trapezoid
No lines of symmetry

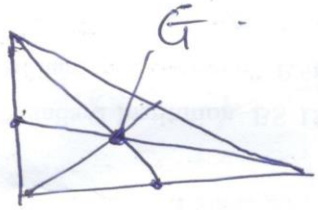


parallelogram
No lines of symmetry

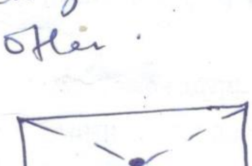


(1) C.G for a Uniform rod lies at its middle point

(2) C.G for a triangle lies at the point where the three medians of the triangle meet.

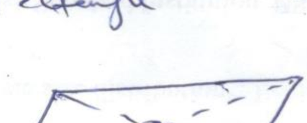


(3) C.G of rectangle, parallelogram is at the point where its diagonals meet each other.

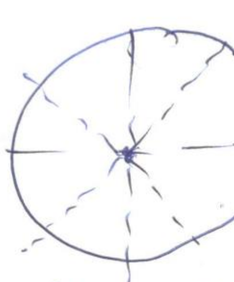


rectangle

square

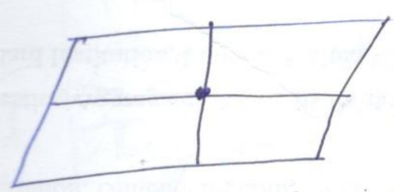


Diagonal.



Since there are an infinite number of lines through the centre, the circle has an infinite number of lines of Symmetry.

It is also the point of intersection of the line joining the middle points of the opposite sides



(4) The C.G of a Circle is at its centre

Pg 4

Unsymmetrical section

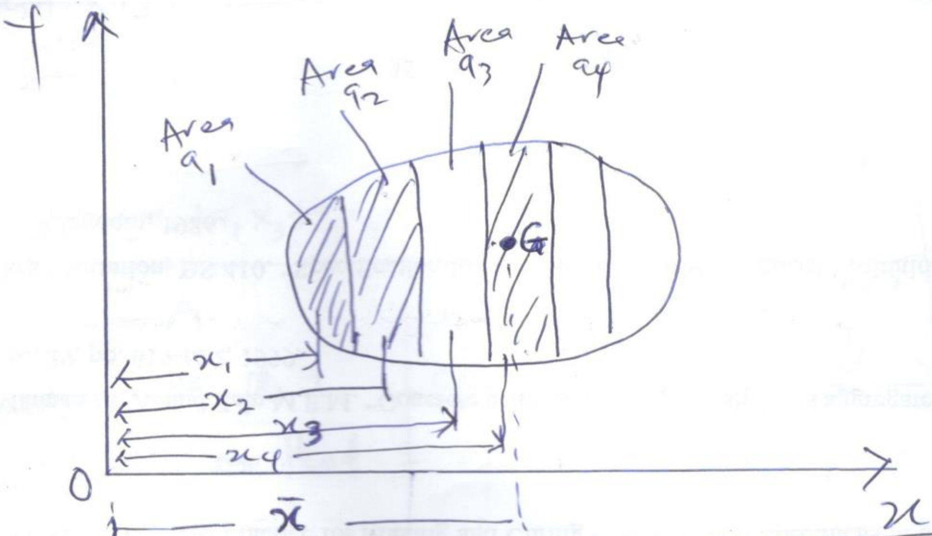


Figure above shows a plane figure of total area A where centre of gravity is to be determined.

let area A composed of a number of small areas

i.e $A = a_1 + a_2 + a_3 + a_4 + \ldots$

let x_1 = distance of CG of area a_1 from OY axis

x_2, x_3, x_4 } Similarly

and so on.

Moments of all small areas about the axis OY

$= a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \ldots$

let G be the centre of gravity of total Area A whose distance from the axis OY is $\bar{x}$

The moment of Total area about $OY = A\bar{x}$

The moment of all small areas about the axis OY must be equal to the moment of total Area about the same axis

Pg5

17

Hence

$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots$

$A\bar{x}$

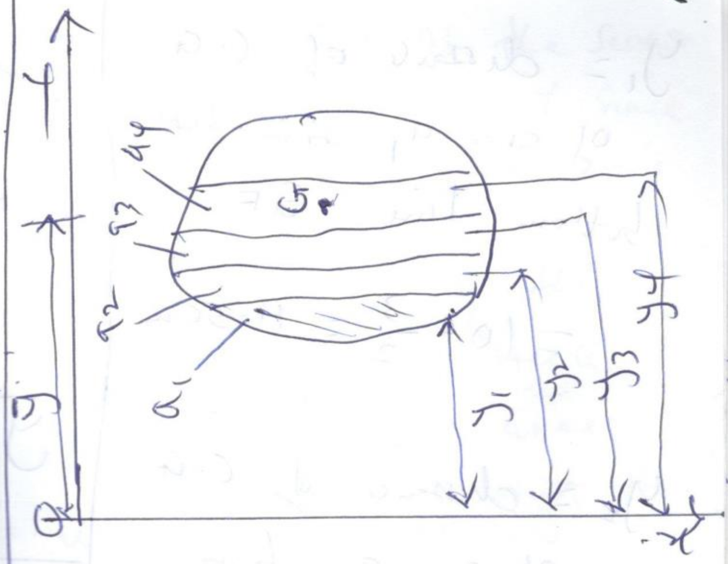
$$\bar{x} = \frac{a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \dots}{A}$$

By Integration Method or Summation method CG of Areas of plane figures is given by

$$\bar{x} = \frac{\sum a_i x_i}{\sum a_i}$$

Taking moments of small areas about the axis OX

$$\bar{y} = \frac{\sum a_i y_i}{\sum a_i}$$



Where

$i = 1, 2, 3, 4, \dots$

x_i = distance of CG of area a_i from axis OY.

y_i = distance of CG of area a_i from axis OX

pg 6

18

y_1 = distance of C G of area a_1 from the bottom line GF

$= 10 + \frac{3}{2} = 11.5$ Cm

y_2 = distance of C.G of area a_2 from bottom line GF

$y_2 = 10/2 = 5$ Cm.

Pg 7

$\bar{y} = \frac{36 \times 11.5 + 30 \times 5}{66}$ $\frac{Cm^3}{Cm^2}$

$= \frac{414 + 150}{66}$

$\bar{y} = 8.545$ Cm.

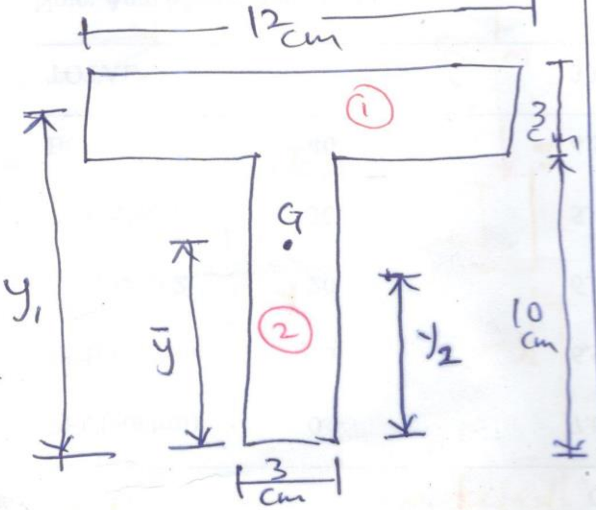
$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{A}$

$= \frac{20 \times 1 + 16 \times 4}{36}$

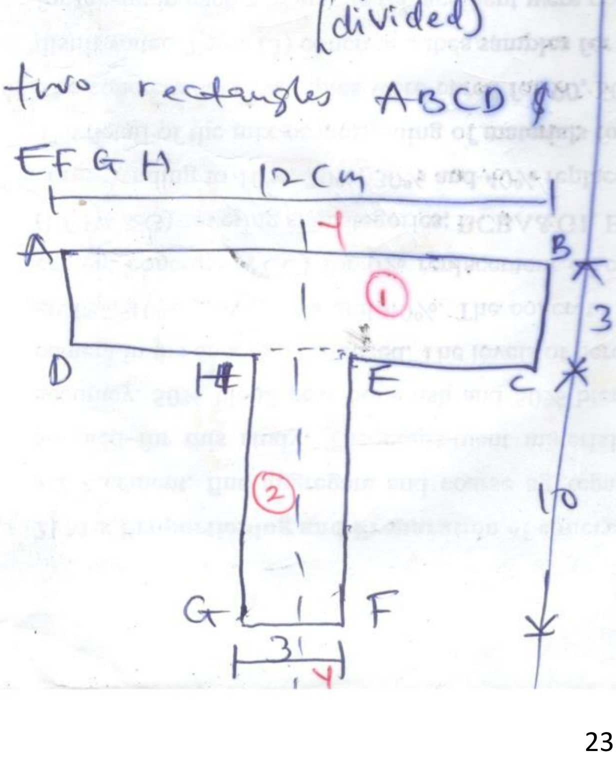
$= \frac{20 + 64}{36} =$

Ex

Find the Centre of gravity of T-Section below.



The T-Section is Symmetrical about axis Y-Y. The Section can also be split into (divided) two rectangles ABCD & EFGH



Hence the C G of the Section will lie on axis Y-Y since it is Symmetrical at axis Y-Y.

The lowest line of the figure is G·F. Hence moments of the area will be taken about the line GF.

$\bar{y}$ = distance of C G of the T-Section from bottom line G·F.

a_1 = area of ▭ ABCD

$= 3 \times 12 = 36\,cm^2$

a_2 = area of ▭ EFGH

$= 10 \times 3 = 30\,cm^2$

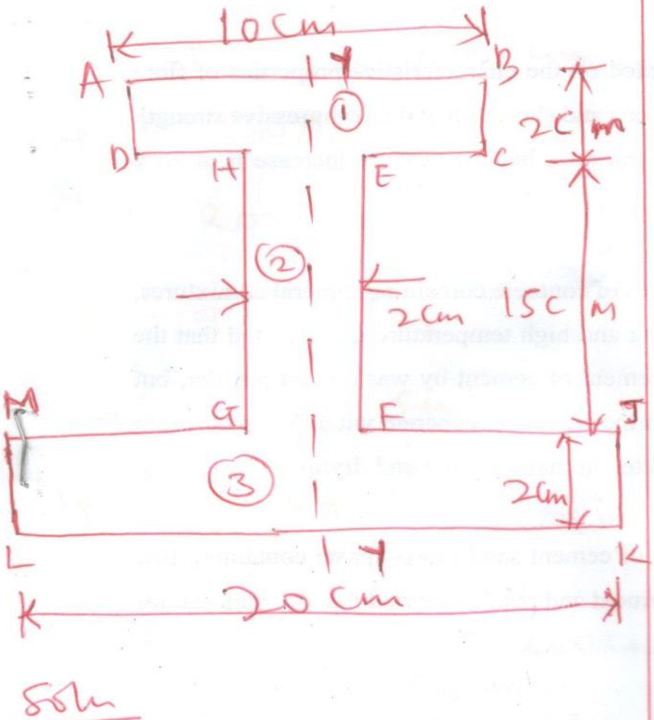
$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A}$$

$A = a_1 + a_2$

$= 66\,cm^2$.

Opposite

Pg 8



Soln

The section is symmetrical about Y-Y axis

a_1 = Area of ▭ ABCD

$= 10 \times 2 = 20\ cm^2$

a_2 = Area of ▭ EFGH

$= 2 \times 15 = 30\ cm^2$

a_3 = Area of ▭ JKLMG

$= 2 \times 20 = 40\ cm^2$

$y_1 = 17 + \frac{2}{2} = 18\ cm$

$y_2 = 2 + \frac{15}{2} = 9.5\ cm$

$y_3 = \frac{2}{2} = 1\ cm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{A}$$

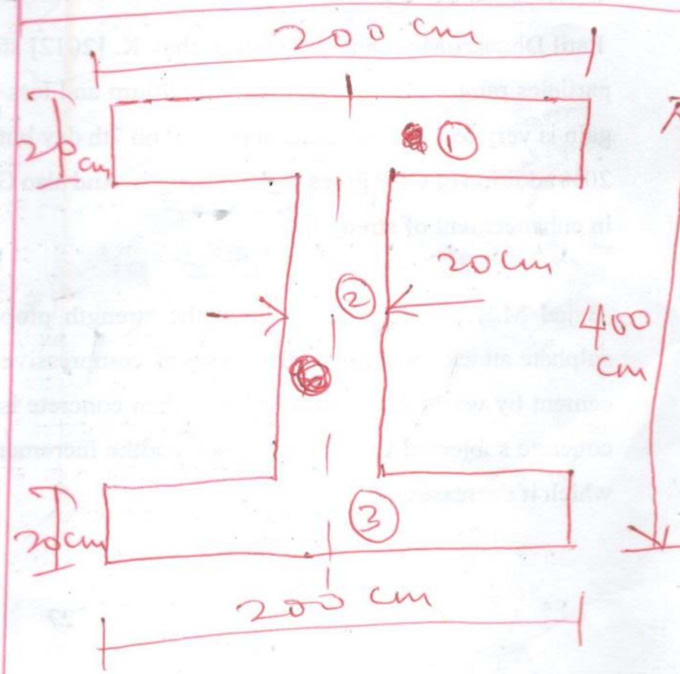
$A = a_1 + a_2 + a_3$

$= 20 + 30 + 40 = 90\ cm^2$

$$\bar{y} = \frac{20 \times 18 + 30 \times 9.5 + 40 \times 1\ (cm^3)}{90\ (cm^2)}$$

$$\bar{y} = \frac{360 + 285 + 40\ cm^3}{90\ cm^2}$$

$\bar{y} = 7.61$ cm from the lowest line LK.



pg 9

$a_1 = 200 \times 2 = 4000\ cm^2$

$a_2 = 360 \times 20 = 7200\ cm^3$

$a_3 = 200 \times 20 = 4000\ cm^3$

$A = a_1 + a_2 + a_3$

$= 15200\ cm^2$

$y_1 = 380 + \frac{20}{2} = 390\ cm$

$y_2 = 20 + \frac{360}{2} = 200\ cm$

$y_3 = 20/2 = 10\ cm$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{A} \quad \frac{(cm^3)}{cm^2}$$

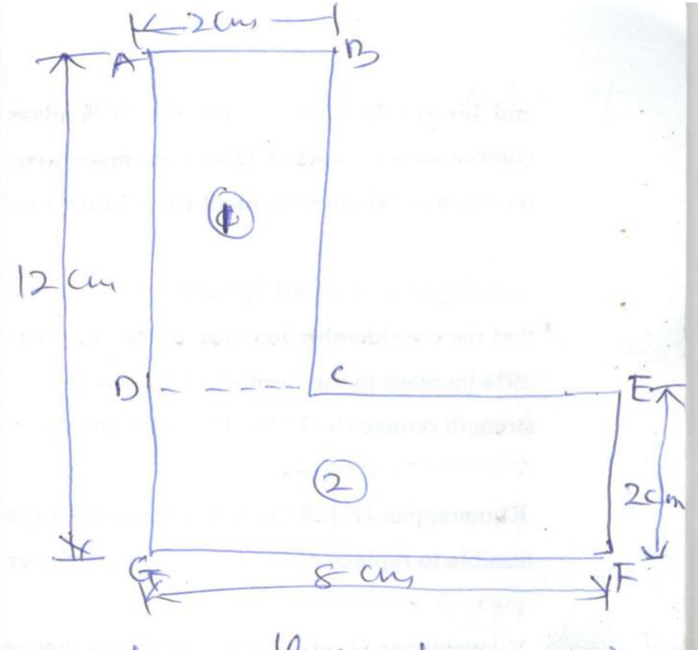
$$= \frac{4000 \times 390 + 7200 \times 20 + 4000 \times 10}{15200}$$

$\bar{y} = 200\ cm$

Merely looking at the I-Section we can tell that CG will lie at $\frac{400}{2}$

$= 200\ cm$

(Pg10)



We know that L-Section is not Symmetrical about any axis.

So we find $\bar{x}$ and $\bar{y}$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{A}$$

$$\bar{y} = \frac{a_1 y_1 + a_2 x_2}{A}$$

$a_1 = 2 \times 10 = 20\ cm^2$

$a_2 = 8 \times 2 = 16\ cm^2$

$y_1 = 2 + \frac{10}{2} = 7\ cm$

$y_2 = 2/2 = 1\ cm$

$x_1 = \frac{2}{2} = 1\ cm$

$x_2 = \frac{8}{2} = 4\ cm$

$A = a_1 + a_2 = 20 + 16 = 36\ cm^2$

(~~35~~)

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{A}$$

$$= \frac{20 \times 1 + 16 \times 4}{36}$$

2.33 cm from left line AG.

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{A}$$

$$= \frac{20 \times 7 + 16 \times 1}{36}$$

$= 4.3$cm from bottom line GF.

PART I

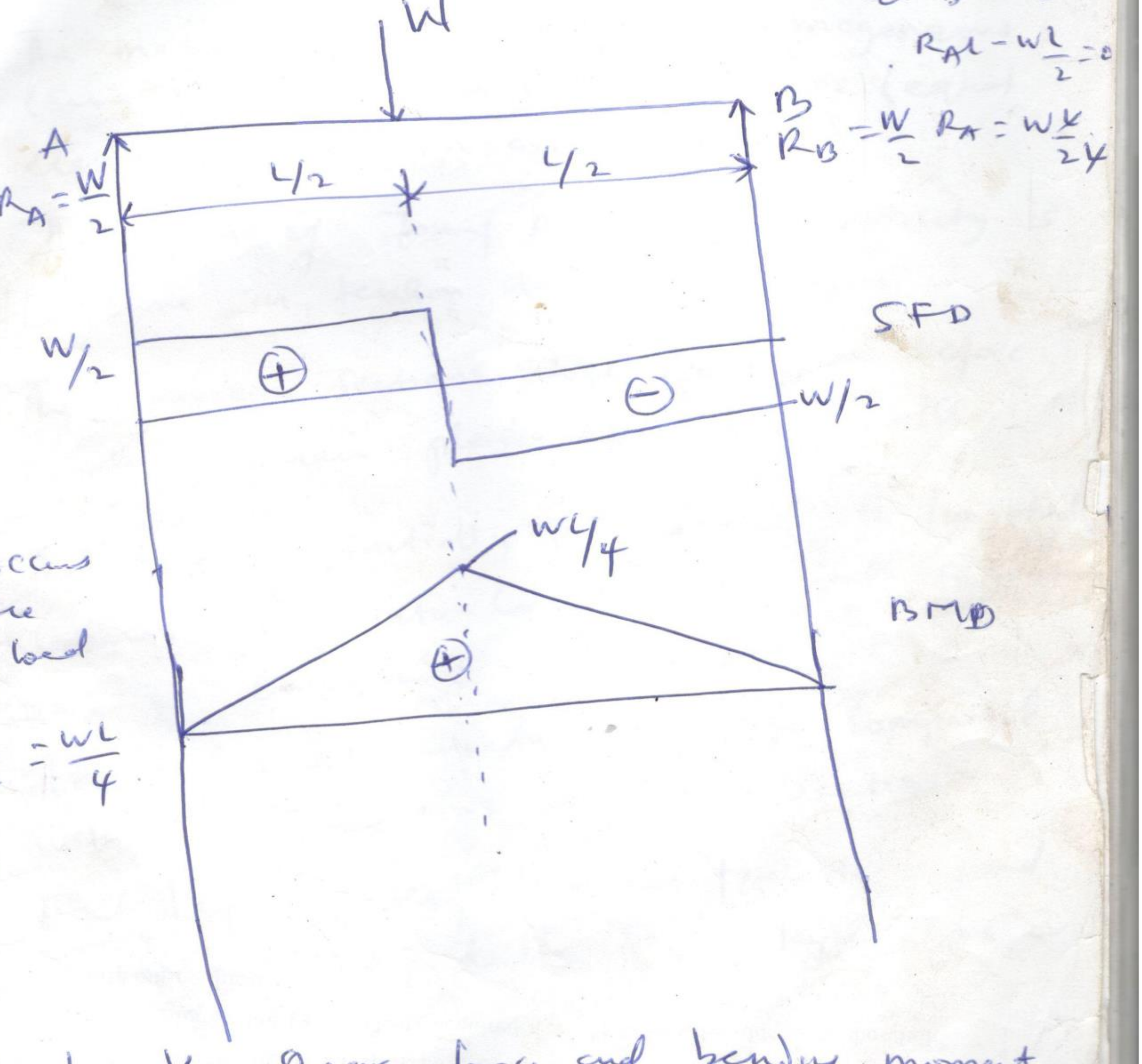
THEORY OF SIMPLE BENDING OF BEAMS

When an external load acts on a beam, the Shear force and bending moment are set up at all sections of the beam.

$\Sigma M_B = 0$

$R_A L - W\frac{L}{2} = 0$

$R_A = \frac{WL}{2L}$



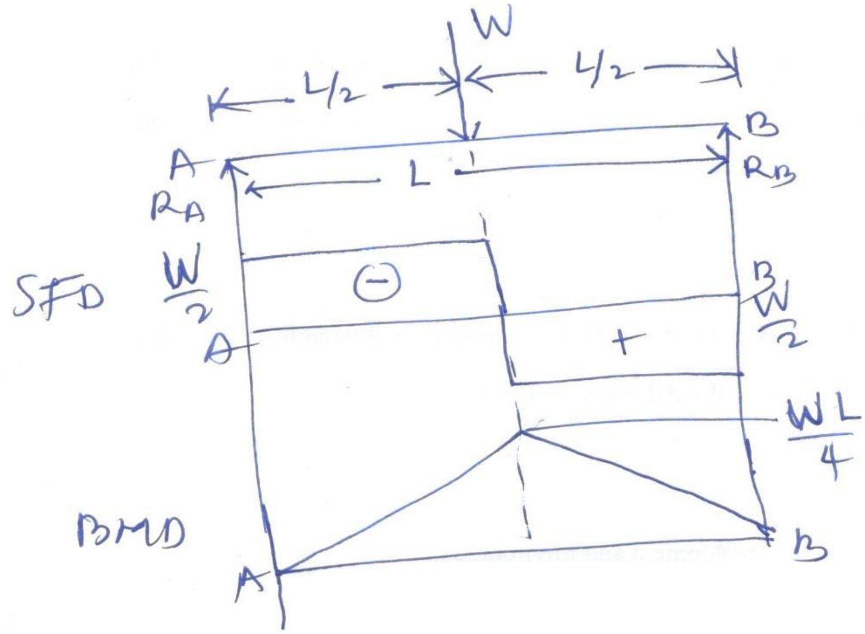
M_{max} occurs under the external load i.e

$\frac{W}{2} \times \frac{L}{2} = \frac{WL}{4}$

Due to the shear force and bending moment, the beam undergoes deformation. The material of the beam will offer resistance or stresses against these deformations. These stresses will

12

Point load



$\Sigma M_B = 0$

$R_A L = \frac{WL}{2}$

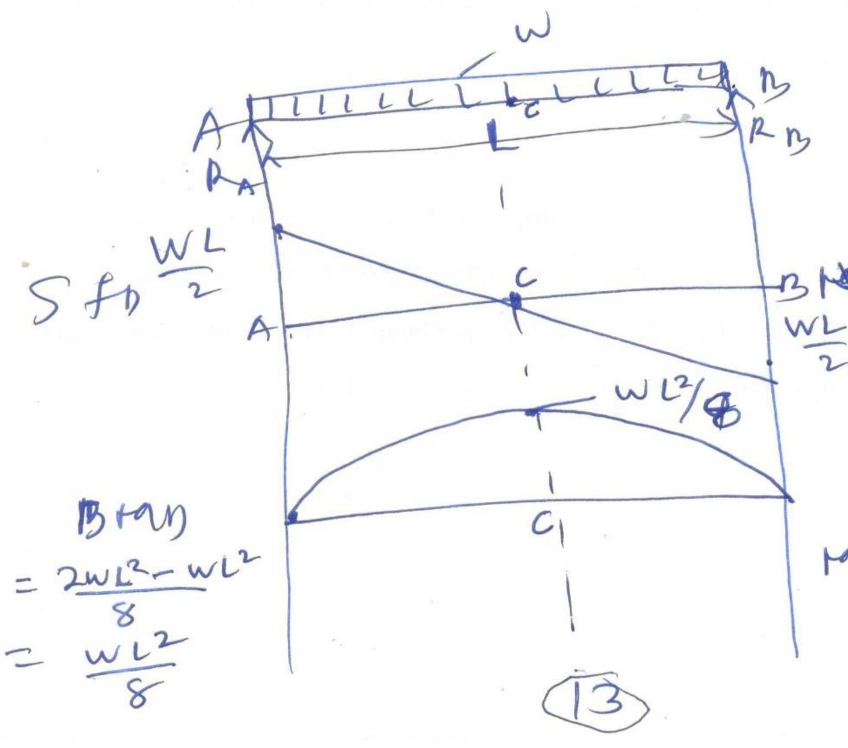
$R_A = \frac{W}{2} = R_B$

M_{max} occurs under point load

$M_{max} = R_A \times \frac{L}{2}$

$= \frac{WL}{4}$

Uniformly distributed load



$\Sigma M_B = 0$

$R_A L = wL \times \frac{L}{2}$

$R_A L = \frac{wL^2}{2}$

$R_A = \frac{wL}{2} = R_B$

$M_C = R_A \frac{L}{2} - \frac{wL}{2} \times \frac{L}{4}$

$= \frac{wL^2}{4} - \frac{wL^2}{8}$ ⇐

$M_C = \frac{2wL^2 - wL^2}{8}$

$= \frac{wL^2}{8}$

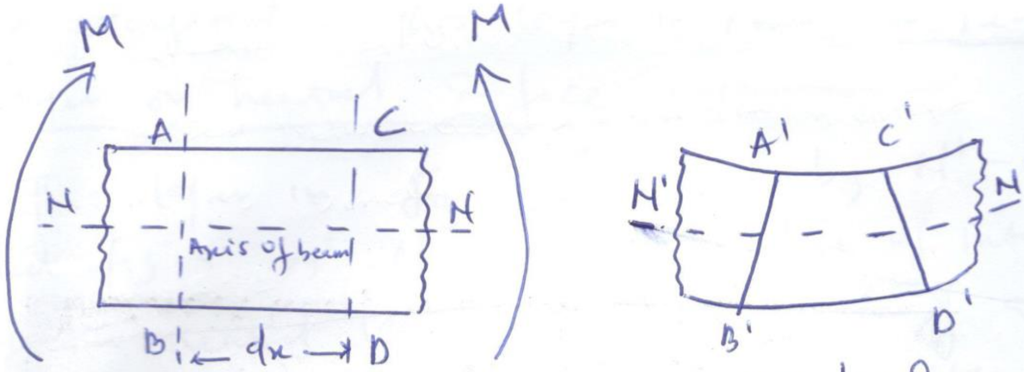
13

with certain assumptions can be calculated. The stresses introduced by bending moment are known as bending stresses.

ASSUMPTIONS OF SIMPLE BENDING

1. The material of the beam is homogeneous (same kind throughout) and isotropic (equal elastic properties in all directions).
2. The value of Young modulus of elasticity is the same in tension and compression.
3. The transverse sections which were plane before bending, remain plane after bending also.
4. The beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature.
5. The radius of curvature is large compared with the dimensions of cross-section.
6. Each layer of the beam is free to expand or contract, independently of the layer, above or below it.

14



(a) Before bending (b) After bending.

Fig (a) shows a part of a beam subjected to simple bending. Consider a small length dx of this part of beam. Consider 2 sections AB & CD which are normal to the axis of the beam N–N. Due to the action of bending moment, the part of length dx will be deformed as shown in fig (b). From fig (b), it is clear that all layers of the beam, which were originally of the same length do not remain the same length anymore. Top layer AC has deformed to A'C'. This layer has been shortened in its length. The bottom layer BD has deformed to B'D'. This layer has been elongated.
From fig B, it is clear that some layers are shortened and some layers are elongated. At a level between top and bottom of the beam, there will be a layer which is neither shortened

15

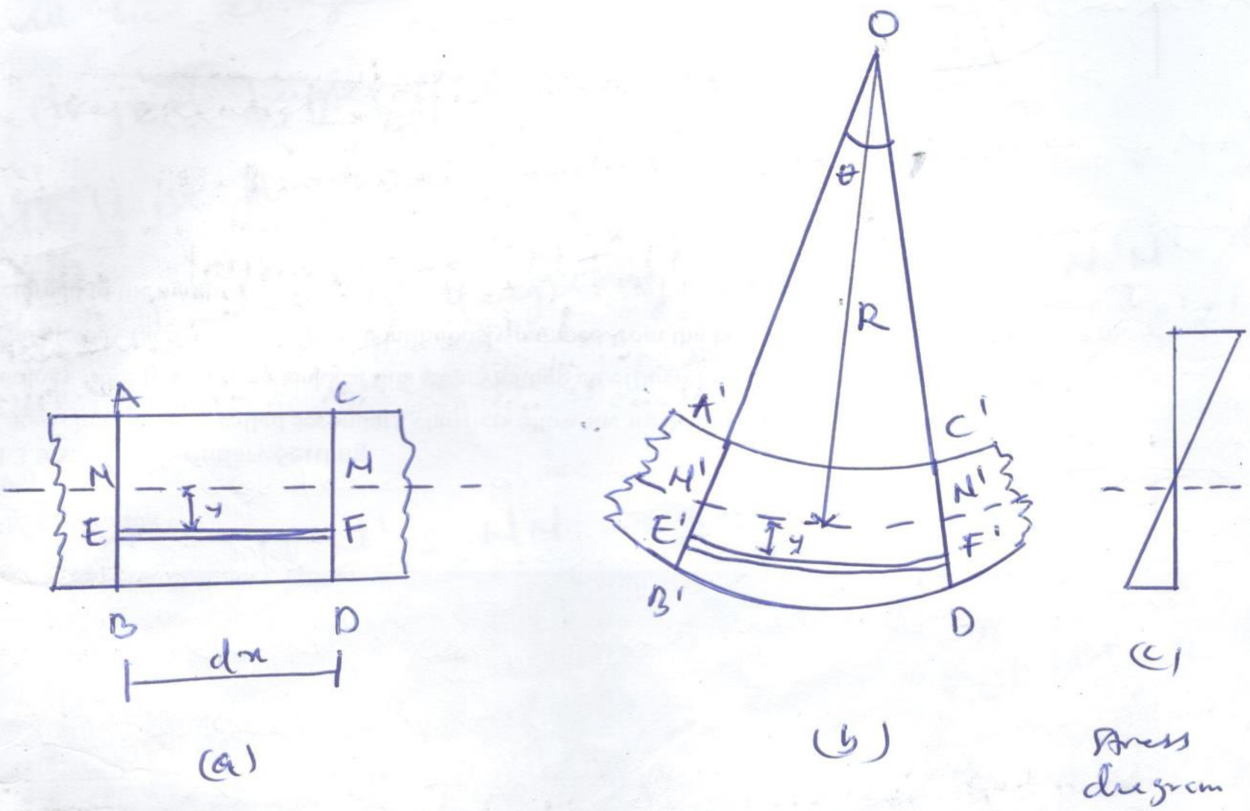
nor elongated. This layer is known as neutral layer or neutral surface.

This layer in fig (b) is shown by N' – N' and fig a by N – N. The line of intersection of the neutral layer on a cross-section of a beam is know as neutral axis (written as NA)

The layers above N – N (or N' – N') have been shortened and those below, have been elongated. Due to the decrease in lengths of the layers above N – N, these layers will be subjected to compressive stresses. Due to increase in the lengths of layers below N – N, these layers will be subjected to tensile stresses.

~~We also see~~ It is also showed that the top layer has been shortened maximum. As we proceed from layer N – N going up, the decrease in length of the layer decreases. At the layer N – N, there is no change in length. This means that compressive stress will be maximum at the top layer. Similarly the increase in length will be maximum at the bottom layer. As we proceed from bottom layer towards the layer N – N, the increase in length of layers decreases. (16) Hence the amount by which a layer increases or decreases in length, depends upon the position of the layer with respect to N – N. This theory of bending is known as Theory of simple bending.

Expression for bending stress



Strain Variation Along the Depth of Beam

fig a shows a small length dx of a beam subjected to a simple bending. Due to action of bending, the part of length dx will be deformed as shown in fig b.

Let $A'B'$ and $C'D'$ meet at O.

let R = Radius of Neutral layer $N'N'$

θ = Angle subtended at O by $A'B'$ & $C'D'$ produced.

(17)

Strain Variation

Consider a layer EF at a distance y below the Neutral layer NM. After bending this layer will be elongated to E'F'

Original length of layer EF = dx

Also length of Neutral layer NM = dx

After bending, the length of neutral layer N'M' will remain unchanged. But length of layer E'F' will increase.

Hence $N'M' = NM = dx$

Now from fig (b),

$N'M' = R \times \theta$ ——— Radius of N'M' = R

and $E'F' = (R+y) \times \theta$ ——— Radius of E'F' = R+y

But $N'M' = NM = dx$

Hence $dx = R \times \theta$

Increase in length of the layer EF

$= E'F' - EF$

$= (R+y) \times \theta - R \times \theta$ — [from the original length $EF = dx = R \times \theta$]

$= (R \times \theta) + y \times \theta - (R \times \theta)$

$= y \times \theta$.

Strain in layer EF $= \dfrac{\text{Increase in length}}{\text{Original length.}} = \dfrac{y \times \theta}{R \times \theta}$

(18)

i.e. Strain in the layer EF $= \frac{y}{R}$

~~As~~ ~~As~~ ~~Is~~ As R is constant, hence strain in layer is proportional to ~~the~~ its distance from the neutral axis.

The equation $\frac{y}{R}$ = strain in layer shows the variation of strain along the depth of the beam. The variation of strain is linear.

Stress Variation

let σ = stress in layer EF

E = Young's modulus of the beam.

Then $E = \frac{\text{stress in layer EF}}{\text{strain in layer EF}}$

$$E = \frac{\sigma}{\frac{y}{R}}$$

i.e. $\sigma = E \times \frac{y}{R}$

$$\sigma = \frac{E}{R} \times y$$

Since E and R are constant, therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer. (19)

The equations $\sigma = \frac{E}{R} \times y$ shows the variation of stress along the depth of the beam. The variation of stress is linear.

In the above case, all layers below the neutral layer are subjected to tensile stresses whereas ~~at~~ the layers above neutral layer are subjected to compressive stresses. Fig c shows the stress distribution

Also $\sigma = \frac{E}{R} \times y$

i.e $\frac{\sigma}{y} = \frac{E}{R}$ • STOP — See examples (8)

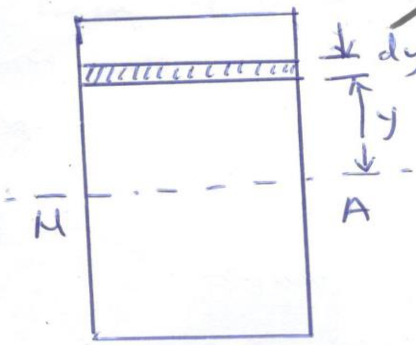
Neutral Axis and Moment of Resistance.

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section. It is written as N.A.

It has been established that if a section is subjected to pure sagging (positive) moment then, the stresses will be compressive at any point above N-A and tensile below the N-A. There is no stress at the N.A.

(20)

The stress at distance y from N.A is given by



$$\sigma = \frac{E}{R} \times y$$

from the cross section of the beam shown, let NA be the neutral axis of the section. Consider a small layer at a distance y from the N.A

let dA = Area of the layer

Now the force on the layer = $\left[\text{Pressure } \text{Stress} = \frac{F}{A}\right]$

= Stress on layer × Area of layer

$= \sigma \times dA$.

$= \frac{E}{R} \times y \times dA$

Total force on the beam section is obtained by integrating the above equation

i.e Total force on the beam

$= \int \frac{E}{R} \times y \times dA$

$= \frac{E}{R} \int y \times dA$

21

But for pure bending, there is no force on the section of the beam. (force is zero).

i.e $\frac{E}{R} \int y \times dA = 0$

$\int y \times dA = 0$ $\left[\text{as } \frac{E}{R} \text{ cannot be zero}\right]$.

$\int y \, dA = 0$

Now $y \cdot dA$ represents the moment area dA about N.A.

Hence $\int y \cdot dA$ represents the moment of entire area of the section about N.A.

But we know that moment of any area about axis passing through its centroid is also equal to zero. Hence N.A coincides with the centroidal axis.

Thus the centroid axis of a section gives the position of N.A.

$$\text{Force on layer} = \frac{E}{R} \times y \times dA$$

Moment of resistance this force about N.A

$$M = \text{Force on layer} \times y$$

$$M = \frac{E}{R} \times y \times dA \times y$$

$$M = \frac{E}{R} \times y^2 \times dA$$

(22)

Total moment of the force on the section of the beam (or moment of resistance)

$$M = \int \frac{E}{R} \times y^2 \times dA$$

$$M = \frac{E}{R} \int y^2 \times dA$$

let M = External moment applied on the beam section.

For equilibrium the moment of resistance offered by the section should be equal to the external bending moment

i.e. $$M = \frac{E}{R} \int y^2 \, dA$$

But the expression $\int y^2 \, dA$ represents the moment of inertia of the area of the section about the neutral axis. Let this moment of inertia be I.

$$M = \frac{E}{R} \times I$$

or $$\frac{M}{I} = \frac{E}{R}$$

But $$\frac{\sigma}{y} = \frac{E}{R}$$

23

1.4 $$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

This equ is applicable to a member which is subjected to a constant bending moment and the member is absolutely free from shear free

This is known as bending equations

It is the most important equ in the theory of simple bending, which gives us relation between various characteristics of a beam.

M = N mm
I = mm^4
σ = N/mm^2
y = mm
E = N/mm^2
R = mm

Check

$$\left.\begin{aligned} \frac{M}{I} &= \frac{N\,mm}{mm^4} = N/mm^3 \\ \frac{\sigma}{y} &= \frac{N/mm^2}{mm} = N/mm^3 \\ \frac{E}{R} &= \frac{N/mm^2}{mm} = N/mm^3 \end{aligned}\right\}$$

24

$$\bar{x} = \frac{a_1x_1 + a_2x_2}{A}$$

$$= \frac{20\times1 + 16\times4}{36}$$

2.33 cm from left line AG

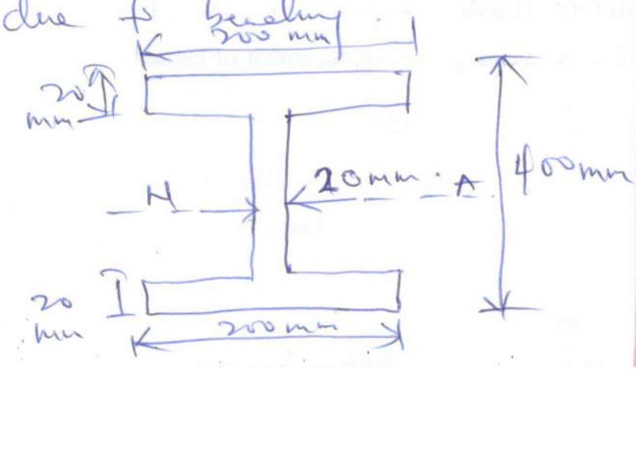
$$\bar{y} = \frac{a_1y_1 + a_2y_2}{A}$$

$$= \frac{20\times7 + 16\times1}{36}$$

≤ 4.3 cm from bottom line GF

Symmetrical

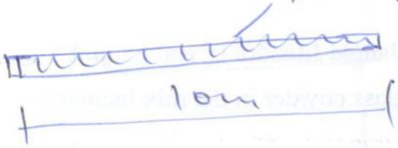
A rolled steel joist of I-Section has dimensions as below: This beam of I-Section carries a udl of 40KN/m run on a span of 10m. Calculate the maximum stress produced due to bending.



Soln

N-A lies at the centre of the I-Section since its symmetrical

$$y_{max} = \frac{400}{2} = 200 \text{ mm}.$$



$$M_{max} = \frac{WL^2}{8} \quad (udl)$$

$L = 10m = 10\times10^3$ mm

$W = 40$ kN/m $= 40,000$ N/m.

$$M_{max} = \frac{40,000\times10^2}{8}$$

$= 500,000$ Nm

Moment of Inertia I

$$I = \frac{bd^3}{12}$$

$$I = \frac{200\times400^3}{12} - \frac{(200-10)\times360^3}{12}$$

$$I = 3.28\times10^8 \text{ mm}^4$$

Pg (25) (25)

Check relation from bending equation and use appropriately.

$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$= \frac{5\times10^8 \times 200}{3.28\times10^8}$$

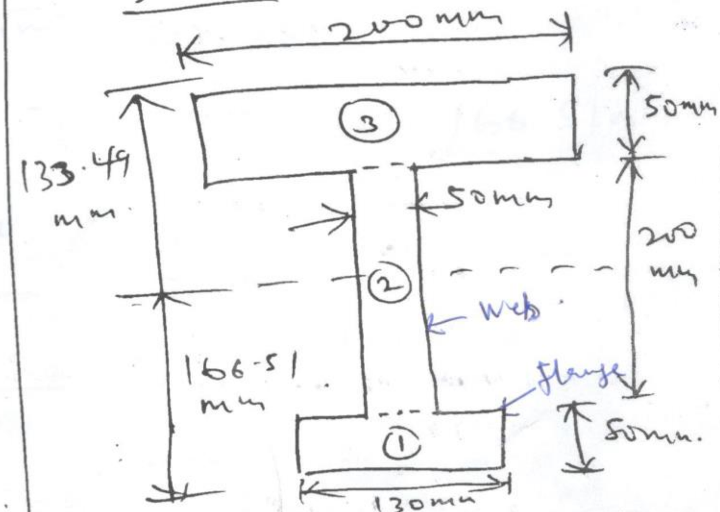
$$\sigma_{max} = 304.87\,N/mm^2$$

Unsymmetrical Section

A Cast iron bracket subjected to bending has cross-section of I-form with unequal flanges as shown in the figure below. Find the position of the neutral axis and moment of inertia of the section about the neutral axis.

If the maximum bending moment on the section is 40 MNmm, determine the maximum stress. What is the nature of the stress?

Soln.



Calculate the CG of the section i.e. y_{max}.

> Note: y = distance of C-G from the bottom line

$a_1 = 130 \times 50 = 6500\,mm^2$
$a_2 = 200 \times 50 = 10{,}000\,mm^2$
$a_3 = 200 \times 50 = 10{,}000\,mm^2$

$y_1 = 50/2 = 25\,mm$
$y_2 = 50 + 200/2 = 150\,mm$
$y_3 = 250 + 50/2 = 275\,mm$

P 926

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2 + a_3 y_3}{A}$$

$$A = a_1 + a_2 + a_3 = 26500 \text{ mm}^2$$

$$= \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{26500}$$

$$\bar{y} = 166.51 \text{ mm}$$

Hence the N.A is at a distance 166.51mm from the bottom face.

(2) $$I = \frac{bd^3}{12}$$

$$I = I_1 + I_2 + I_3$$

I_1 = MOI of the bottom flange about N.A

= MOI of the bottom flange about an axis passing through its C.G. + $\left[A_1 \times (\text{Distance of its C.G from N.A})^2 \right]$

$$I_1 = \frac{130 \times 50^3}{12} + 6500 (166.51 - 25)^2$$

$$= 1354166.67 + 130163020$$

$$= 131517186.6 \text{ mm}^4$$

~~24~~ Pg 27

I_2 = M.O.I of web about N.A

$$= \frac{50 \times 200^3}{12} + A_2 (166.51 - 150)^2$$

$$= 33333333.33 + 10,000 (166.51 - 150)^2$$

$$+ 272580.1$$

$$= 36059134.33 \text{ mm}^4.$$

I_3 = M.O.I of the top flange about N.A

$$= \frac{200 \times 50^3}{12} + 1000 (y_3 - 166.51)^2$$

$$= 2083333.33 + 10000 (275 - 166.51)^2$$

$$= 119784134.3 \text{ mm}^4.$$

$$I = I_1 + I_2 + I_3$$

$$= 131517186.6 + 3$$

$$= 284907234.9$$

(2) (3) (28)

Now distance of C G from the top fibre

$$= 300 - \bar{y} = 300 - 166.51$$

$$= 133.49\,mm$$

and distance from bottom fibre is $\bar{y} = 166.51\,mm$. —— (bigger)

Hence we shall take the value of $y_{max} = 166.51\,mm$ for maximum bending stress.

Now using equation.

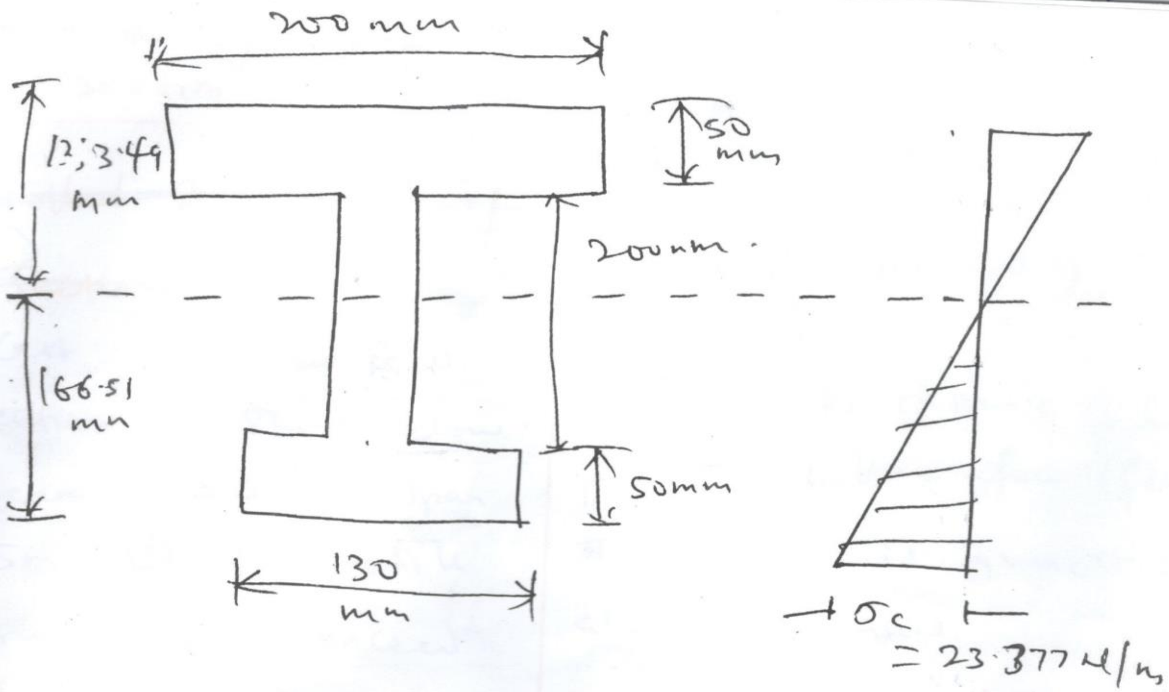
$$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

$$\sigma_{max} = \frac{M}{I} \times y_{max}.$$

$$= \frac{40 \times 10^6}{284907234.9} \times 166.51$$

$$= 23.277\,N/mm^2$$

(4) 29



The stress is compressive because It is a cantilever (bracket)

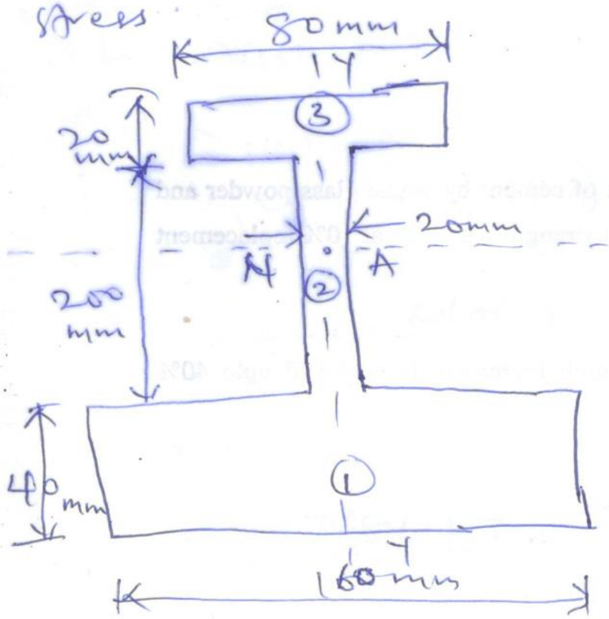
In case of cantilevers, upper layer is subjected to tensile stress, whereas lower layer is subjected to compressive stress.

(5) (30)

Examples on ~~Exam~~ UnSymmetrical Section

~~A rolled steel joint of I-Section~~

A Cast iron beam is of I-Section as shown below. The beam is SS on a span of 5m. If the tensile stress is not to exceed $20\,N/mm^2$. Find the safe uniform load which the beam can carry. Find also the maximum compressive stress



Soln

Length of beam $L = 5m$

Max tensile stress $\sigma_t = 20\,N/mm^2$

First, calculate the CG of the section $\bar{y}$

Let $\bar{y}$ is the distance of CG from the bottom face (flange)

The section is symmetrical about Y-axis

$A_1 = 160 \times 40 = 6400\,cm^2$

$A_2 = 200 \times 20 = 4000\,cm^2$

$A_3 = 80 \times 20 = 1600\,cm^2$

$y_1 = 40/2 = 20$

$y_2 = 40 + 200/2 = 140$

$y_3 = 240 + 20/2 = 250$

$A = A_1 + A_2 + A_3$
$= 12000\,mm^2$

$\bar{y} = \dfrac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$

$= \dfrac{1088000}{12000}$

$= 90.66\,mm$

~~28~~ 31

N·A lies at a distance of 90·66mm from the bottom face or

$260 - 90.66 = 169.34$ mm

from the Top face.

Now

M.O.I of the section about N-axis is given by $I = I_1 + I_2 + I_3$

I_1 = M.O.I of bottom flange about N.A

= M.O.I of bottom flange about its C.G + $\left[A_1 \times \left(\text{Distance of its C.G from N.A}\right)^2\right]$

$= \frac{160 \times 40^3}{12} + 6400(90.66 - 20)^2$

$= 853333.33 + 31954147.84$

$= 32807481.17 \text{ mm}^4$

I_2 = M.O.I of web about NA

= M.O.I of web about its C.G $+ \left[A_2 \times \left(\text{Distance of its C.G from N.A}\right)^2\right]$

$= \frac{20 \times 200^3}{12} + 4000(140 - 90.66)^2$

$= 23071075.75 \text{ mm}^4$

I_3 = M.O.I of top flange about N.A

= M.O.I of top flange about its C.G $+ \left[A_3 \times \left(\text{Distance of its C.G from N.A}\right)^2\right]$

$= \frac{80 \times 20^3}{12} + \overset{1600}{\cancel{8000}}(250 - 90.66)^2$

$= 40676110.29 \text{ mm}^4$

$I = I_1 + I_2 + I_3$

$= 96554667.21 \text{ mm}^4$

For a simple supported beam, the tensile stress will be at the extreme bottom fibre and the compressive ~~force~~ stress will be at the extreme top fibre

32

Here, Max tensile stress

$\sigma_t = 20\ N/mm^2$

$y_{max} = 90.66\ mm$

i.e. y_{max} is the distance of the extreme ~~top~~ bottom fibre (where the tensile stress is maximum) from the N.A

Using the relevant bending equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$M = \frac{\sigma}{y} \times I$$

$$= \frac{20}{90.60} \times 96554667.21$$

$$= 21300389.85\ Nmm$$

let w = udl in N/m on the SSB

The max B.M is at the centre and equal to

$$\frac{wl^2}{8}$$

i.e. $$M_{max} = \frac{w \times 5^2}{8}$$

$$= \frac{w \times (25 \times 1000\ mm)}{8}$$

$$M_{max} = 3125w$$

i.e. $21300389.85 = 3125\ w$

$$w = \frac{21300389.85}{3125}$$

$$= 6816.125\ N/mm$$

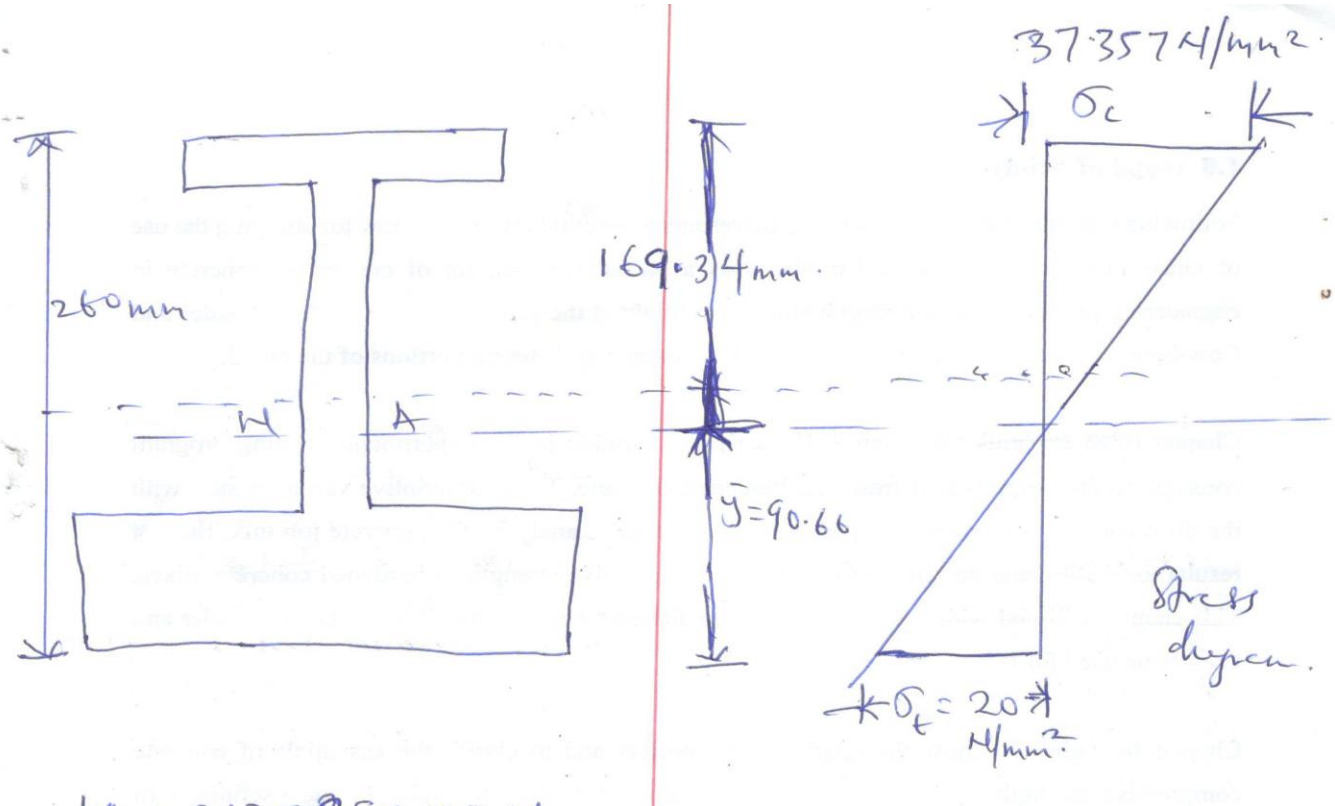
Maximum Compressive Stress

Distance of the extreme top fibre from N.A

$y_c = 260 - 90.66$

$= 169.34\ mm$

~~30~~ 33



$M = 21300389.85$ N mm

$I = 965546 67.21 \text{ mm}^4$.

σ_c = Max compressive Stress

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \times y$$

$$\sigma_c = \frac{M}{I} \times y_c$$

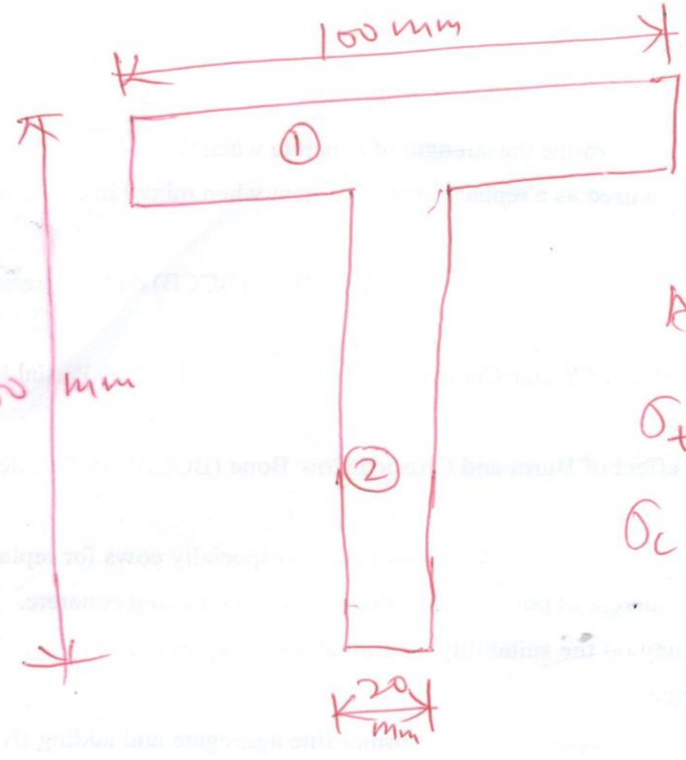
$$\sigma_c = \frac{21300389.85}{965546667.21} \times 169.34$$

$$= 37.357 \text{ N/mm}^2$$

34

Home work

A Cast Iron beam is of T-Section as below. The beam is SS on a span of 8 m. The beam carries a ~~load~~ udl load of 1.5 kN/m length on the entire span. Determine the maximum tensile and compressive stresses.



Answer

$\sigma_t = 258.81\ N/mm^2$

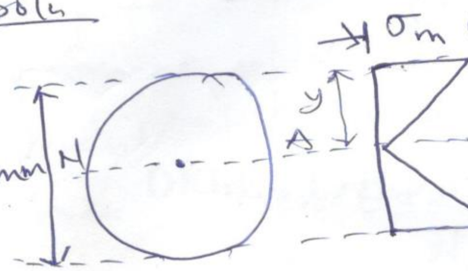
$\sigma_c = 123.08\ N/mm^2$

35

Examples (SOM RAJPUT Pg 347)

1) A steel wire 5mm θ is bent into a circular shape of 5m radius. Det. the max. stress induced in the wire. Take $E = 200$ GPa

Soln



θ of the wire $= 5mm$

Radius of circular shape $= 5m = 5\times10^3 mm$

$E = 200\ GPa = 200\times10^3\ N/mm^2$

The distance b/w the N.A wire and its extreme fibre is given as

$$y_m = \frac{d}{2} = \frac{5}{2} = 2.5mm$$

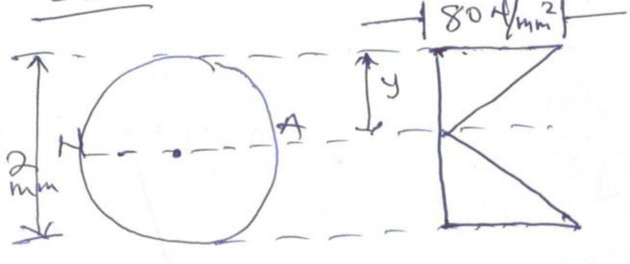
$$\sigma_{max} = \frac{E}{R}\times y = \frac{200\times10^3}{5\times10^3}\times2.5$$

$$\sigma_{max} = 100\ N/mm^2$$

(36)

2) A copper wire of 2mm θ is required to be wound around a drum. Find the min. radius of the drum, if the stress in the wire is not to exceed $80\ N/mm^2$. Take $E = 100$ GPa $[100\times10^3\ N/mm^2]$

Soln



$y_m = \frac{2}{2} = 1mm$

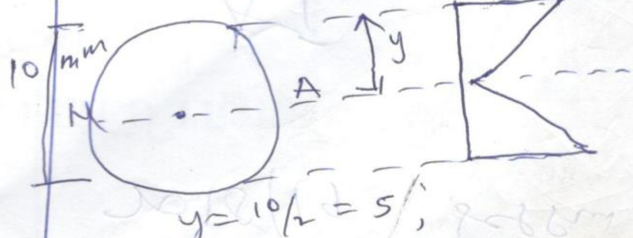
$\sigma = 80\ N/mm^2$

$E = 100\times10^3\ N/mm^2$

$80 = \frac{100\times10^3}{R}\times1$

$R = 1.25\times10^3 mm = 1.25m$

3) A metallic rod of θ 10mm is bent into a circular form of radius 4M. σ_{max} developed in the rod is $125\ N/mm^2$. Find the val. of Young modulus of elasticity of the rod material



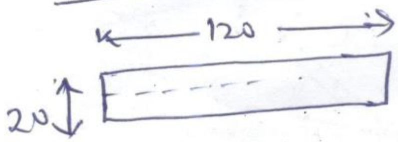
$y = 10/2 = 5$

$E = 100\times10^3\ N/mm^2$

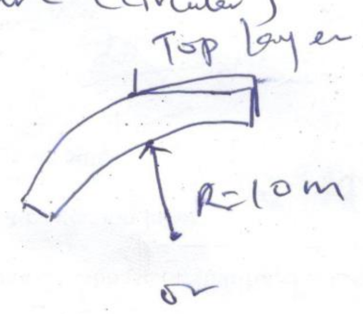
Ex 1

A steel plate of width 120mm and thickness 20mm is bent into a Circular arc of radius 10m. Det. the max. stress Induced and the bending moment which will produce the maximum stress : Take $E = 2 \times 10^5$ N/mm^2

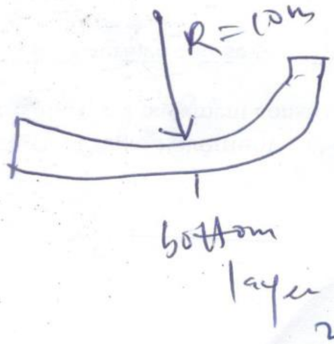
Soln



bent : either to arc (circular)



or



37

width of plate $b = 120mm$; thickness of plate $t = 20mm$ (depth)

moment of Inertia $= \frac{bd^3}{12} = \frac{bt^3}{12}$

$$I = \frac{120 \times 20^2}{12} = 8 \times 10^4 mm^4$$

Radius of curvature $R = 10m$
$= 10 \times 10^3 mm$

$E = 2 \times 10^5 N/mm^2$ [What is E?]

let σ_{max} = Max stress Induced

M = Bending moment.

Using the Relevant bending equation

i.e $\frac{\sigma}{y} = \frac{E}{R}$

$$\sigma = \frac{E}{R} \times y \quad \text{— (1)}$$

equation 1 gives the stress at a distance y from N.A.

Stress will be Max, when y is maximum. But y will be max. at ~~top~~ the top layer or bottom layer.

$y_{max} = \frac{t}{2} = \frac{20}{2} = 10mm$

$\sigma_{max} = \frac{E}{R} \times y_{max}$

$= \frac{2 \times \cancel{10^5}^{10^2}}{\cancel{10} \times \cancel{10^3}} \times \cancel{10}$

$= 200 \; N/mm^2$

Bending moment
Using relevant bending eqn

$\frac{M}{I} = \frac{\sigma}{y}$

$M = \frac{\sigma}{y} \times I$

$= \frac{200}{\cancel{10}} \times 8 \times 10^{\cancel{4}\,3}$

$= 16 \times 10^5 \, Nm$

$= 1.6 \times 10^6 \, kNm$

or $\frac{M}{I} = \frac{E}{R}$

$M = \frac{E}{R} \times I$

$= \frac{2 \times 10^5}{10 \times 10^3} \times 8 \times 10^4 = 1.6 \times 10 \, kNm$ (as before)

Ex 2

Cal. the σ_m induced in a cast iron pipe of external diameter 40mm, of internal diameter 20mm and length 4m when the pipe is supported at its ends and carries a point load of 80N at its centre.

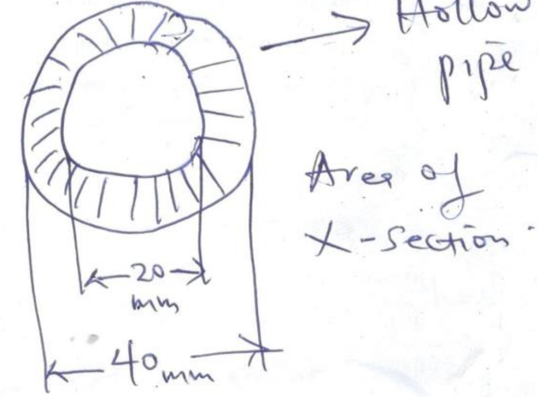
Soln

Ext θ $D = 40mm$

In θ $d = 20mm$

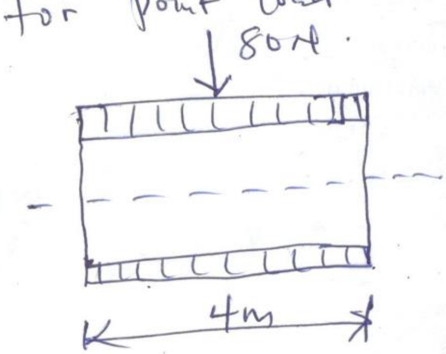
length $L = 4m = 4000mm$

Point load $W = 80N$



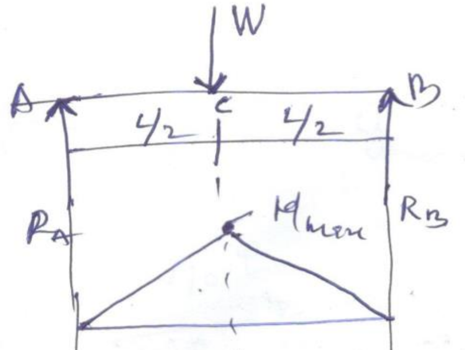
Area of X-section

For point load



$M_{max} = \frac{WL}{4}$ how?

38



$\Sigma M_B = 0$; $R_A L - \frac{WL}{2} = 0$

$R_A = R_B = \frac{W}{2}$

M_{max} at $C = R_A \times \frac{L}{2}$

$= \frac{W}{2} \times \frac{L}{2} = \frac{W}{4}$

i.e. $M_{max} = \frac{80 \times 4000}{4}$

$= 80 \times 10^3$ Nmm

$= 8 \times 10^4$ Nmm.

Moment of Inertia for a Circular pipe; $I = \frac{\pi d^4}{64}$

For Record

I for rectangular Section $I = \frac{bd^3}{12}$

d = depth.

This is hollow pipe

I for hollow pipe

$I = \frac{\pi}{64}(D^4 - d^4)$

$= \frac{\pi}{64}(40^4 - 20^4)$

$= 117809.7 \text{ mm}^4$

Relevant Bending eqn.

$\frac{M}{I} = \frac{\sigma}{y}$

$\sigma = \frac{M}{I} \times y$

~~When σ is max~~;

for max σ; y has to be maximum

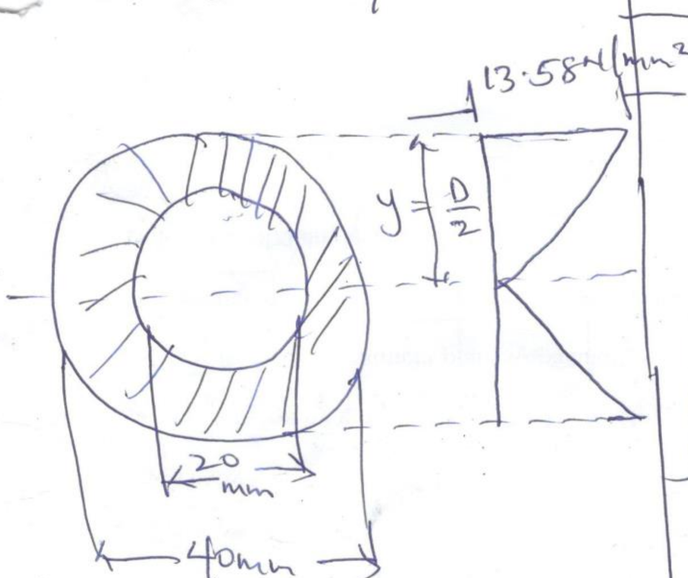
i.e. $y = \frac{D}{2} = \frac{40}{2} = 20$ mm

39 8 17

$$\sigma_{max} = \frac{M}{I} \times y_{max}$$

$$= \frac{8 \times 10^4}{117809.7} \times 20$$

$$= 13.58\ N/mm^2$$



SECTION MODULUS (Z)

It is also called modulus of a section. It is defined as the the ratio of moment of inertia of a section about n.a. to the distance of the outermost layer from the n.a.

Mathematically

$$Z = \frac{I}{y_{max}}$$

But from Bending equation

$$\frac{M}{I} = \frac{\sigma}{y}$$

Stress σ is max when y is max

i.e. $$\frac{M}{I} = \frac{\sigma_{max}}{y_{max}}$$

i.e. $$M = \sigma_{max} \cdot \frac{I}{y_{max}}$$

i.e. $$M = \sigma_{max} Z$$

i.e. $$\sigma_{max} = \frac{M}{Z}$$

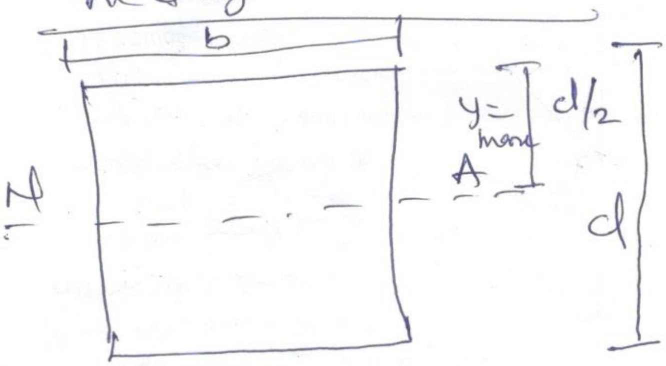
~~M = moment~~ M is The M_{max} or moment of resistance offered by the section.

(40) (9)

Hence from $M = \sigma_{max} Z$

Moment of resistance M is maximum when Z is maximum. Hence Section modulus Z represents the strength of the section.

Z for Rectangular Section



$$I = \frac{bd^3}{12}$$

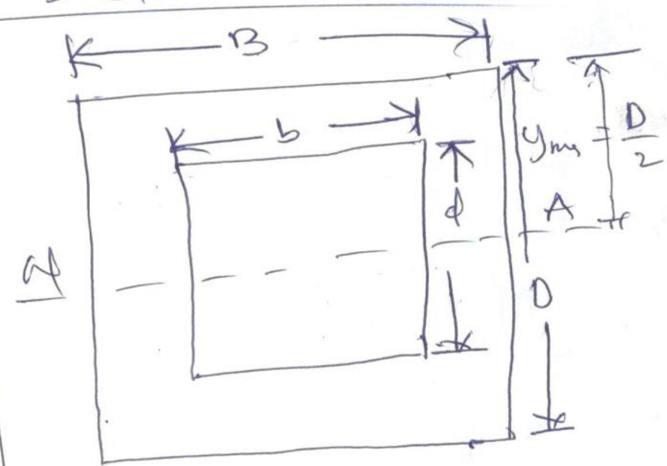
$$y_{max} = \frac{d}{2}$$

Hence

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{12} \div \frac{d}{2}$$

$$Z = \frac{bd^{\cancel{3}\,2}}{6\,\cancel{12}} \times \frac{\cancel{2}}{\cancel{d}} = \frac{bd^2}{6}$$

Z for rectangular Hollow section



$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}\left[BD^3 - bd^3\right]$$

$$y_{max} = \frac{D}{2}$$

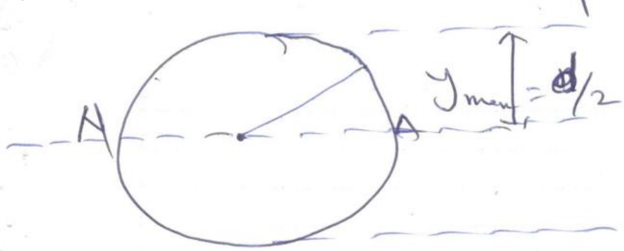
$$Z = \frac{1}{12}\left[BD^3 - bd^3\right] \times \frac{2}{D}$$

$$Z = \frac{BD^3 - bd^3}{6D}$$

41

(18)

Circular Section



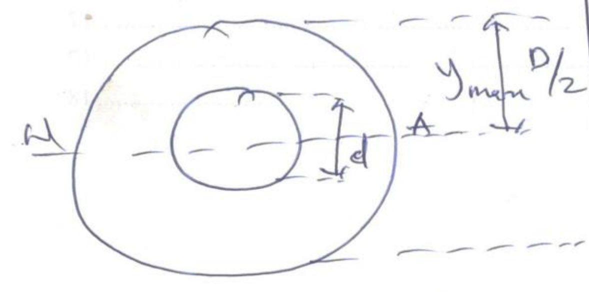
$$I = \frac{\pi d^4}{64}$$

$$y_{max} = \frac{d}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\pi d^{\cancel{4}\,3}}{\cancel{64}_{32}} \times \frac{\cancel{2}}{\cancel{d}}$$

$$Z = \frac{\pi d^3}{32}$$

Circular Hollow



$$I = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} \;; \; y = \frac{D}{2}$$

$$Z = \frac{\pi D^4}{64} - \frac{\pi d^4}{64} \times \frac{2}{D}$$

$$= \frac{\pi}{\cancel{64}_{32}} \left[D^4 - d^4\right] \times \frac{\cancel{2}}{D}$$

$$Z = \frac{\pi}{32D} \left[D^4 - d^4\right].$$

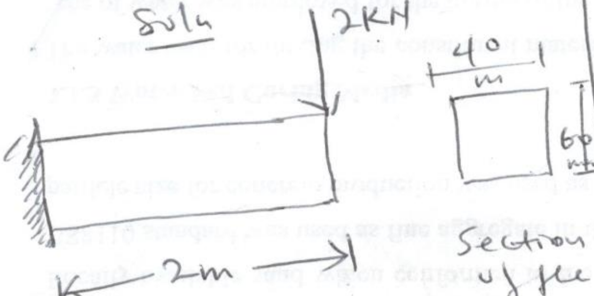
42 20

Example

A bracket (Cantilever) of length 2m having a load of 2 KN at its free end

A Cantilever beam of rectangular section 40mm × 60mm, length 2m having a load of 2KN at its free end. Calculate the maximum stress induced in the beam.

Soln



Section of the beam.

Section modulus of a rectangular section is given by

$$Z = \frac{bd^2}{6}$$

$b = 40\,mm$, $d = 60\,mm$.

$$Z = \frac{40 \times 60^2}{6}$$

$$= 24000\,mm^3$$

Maximum bending moment for a Cantilever beam is at the fixed end.

$$M_{max} = WL$$

$$W = 2\,KN = 2000\,N$$

$$L = 2m = 2000\,mm$$

$$M_{max} = 2000 \times 2000$$

$$= 4 \times 10^6\,Nmm$$

Maximum stress or stress at failure

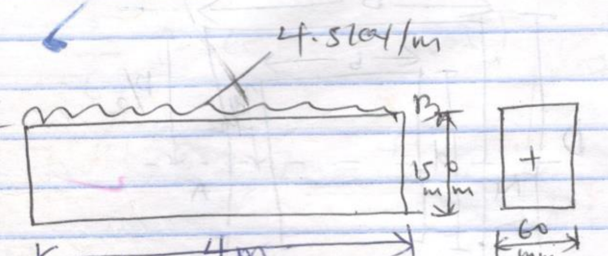
$$\sigma_{max} = \frac{M_{max}}{Z}$$

$$= \frac{4 \times 10^6}{24000}$$

$$\sigma_m = 166.67\,N/mm^2$$

43

A rectangular beam 60mm wide and 150mm deep is SS over a span of 4m. If the beam is subjected to UDL of 4.5KN/m. Find the maximum bending stress induced in the beam.



We know that Z for a rectangular section.

$$Z = \frac{bd^2}{6}$$

$$= \frac{60\times(150)^2}{6}$$

$225\times10^3\,mm^3$ ~~$337.5\times10^3\,mm^3$~~

M_{max} for SSB subjected to udl load.

$$M_{max} = \frac{WL^2}{8} = \frac{4.5\times(4\times10^3)^2}{8}$$

$$= 9\times10^6\ N\,mm$$

$$\sigma_{max} = \frac{M}{Z}$$

$$= \cancel{\frac{9\times10^6}{333.5\times10^3}} \quad \frac{9\times10^6}{225\times10^3}$$

~~$=40N/mm^2$~~ ~~$26.97N/mm^2$~~

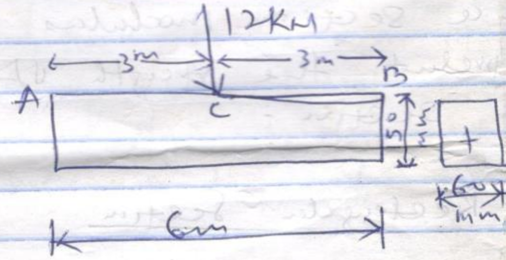
or ~~40MPa~~ $40N/mm^2$.

$= 40MPa$

(2)

A rectangular beam 60mm wide and 150mm deep is simply supported over a span of 6m. If the beam is subjected to central point load of 12KN, find the maximum bending stress induced in the beam section.

Solution



We know that Z for rectangular section

$$Z = \frac{bd^2}{6}$$

$$\frac{60\times(150)^2}{6}$$

$$= 225\times10^3\,mm^3$$

M_{max} for SSB with point load at the centre of the beam

$$M_{max} = \frac{WL}{4}$$

$$= \frac{12\times10^3 \times 6\times10^3}{4}$$

$$= 18\times10^6\ N\,mm.$$

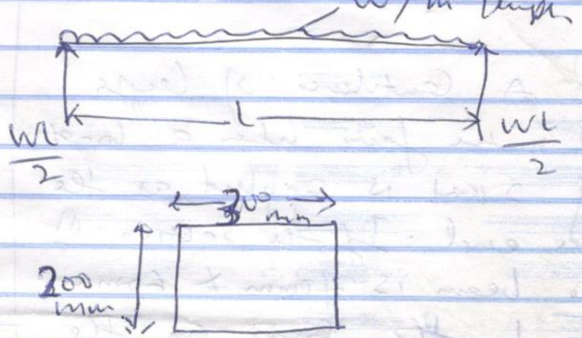
B36 BS 14 Zu

Maximum bending stress induced

$$\sigma_{max} = \frac{M}{Z} = \frac{18 \times 10^6}{225 \times 10^3} = 80N/mm^2$$

$= 80MPa$

(3) A rectangular beam 200mm deep and 300mm wide is SS over a span of 8m. What UDL per metre the beam may carry, if the bending stress is not to exceed $120 N/mm^2$.



Soln

Given

Depth of beam $d = 200mm$

Width of beam $b = 300mm$

$\boxed{\text{length of beam } L = 8m \quad L = 8000mm}$

Max bending stress

$\sigma_{max} = 120N/mm^2$

let w = udl per mm/length over the beam.

$1000 N = 1KN$

$x = 8000,000 KN$

Section modulus $Z = \frac{bd^2}{6}$

$= \frac{300 \times 200^2}{6} = 2 \times 10^6 mm^3$

M_{max} for UDL, $\frac{wl^2}{8} = \frac{w8^2}{8}$

$M = \frac{wl^2}{8} = \frac{w \times (8000)^2}{8}$

$M = w8000\,000$ ~~KN/mm~~ KN.mm $= 8w$ Nm

$M = w8000$ N mm $= w\,8000 N.mm$

$\sigma_{max} = \frac{M}{Z}$

$M = Z \cdot \sigma_{max}$

$4500 N/m = \frac{4500}{1000}\frac{N}{mm} = 4.5N/mm$

$4.5kN/m = 4500N/m$

10mm – 1cm
10 cm – 1dcm
10 dcm – 1m

1000 mm = 1m.

1.0

$8000w = 120 \times 2 \times 10^6$

$w = \frac{\cancel{120}^{30} \times \cancel{2}^{1} \times 10^{\cancel{6}3}}{\cancel{8000}_{4}}$

$w = 30 \times 1000$ N/mm

$w = 30$ kN/mm.

(4) A Cantilever of length 2 metre fails when a load of 2kN is applied at the free end. If the section of the beam is 40mm × 60mm. Find the stress at the failure

Soln

Given

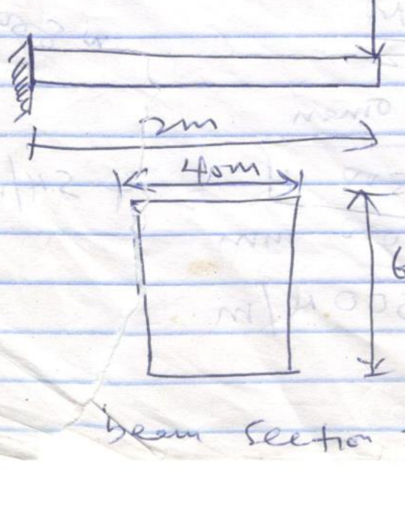
length $L = 2m = 2 \times 10^3$ mm

load $W = 2kN = 2000$ N

Section of beam is 40mm × 60mm

width $b = 40$mm

depth $= 60$mm



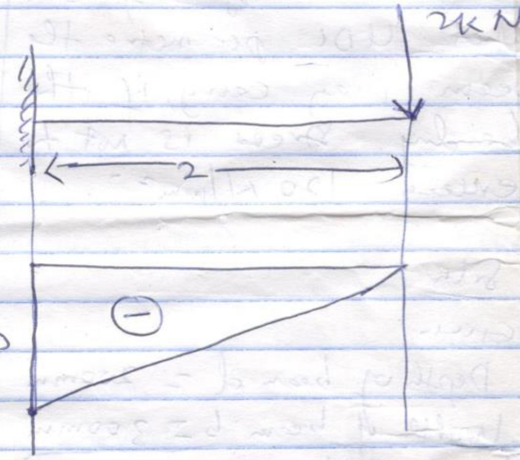
beam section.

$\sigma_{max} = \frac{M_{max}}{Z}$

$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6}$.

$= 2.4 \times 10^{4}$ mm^3

M_{max} for a Cantilever beam as shown Below



$M_{max} = W \times L$

$= 2 \times 10^3 \times 2 \times 10^3$

$= 4 \times 10^6$ N mm

$\sigma_{max} = \frac{4 \times 10^{\cancel{6}2}}{2.4 \times 10^{\cancel{4}}}$

$\sigma_{max} = 1.6 \times 10^2$ N/mm^2.

$= 166.6$ N/mm^2.

CVE ? — Strain — To pull, draw, or stretch tight

CVE 307 Tuesday 10/4/2018.

STRAIN ENERGY -

When a body is strained, energy is absorbed in that body. This energy is called Strain Energy.

The straining effect may be due to gradually applied load or suddenly applied load or load with impact.

Hence the strain energy will be stored in the body when the load is applied gradually or suddenly or with an impact.

The strain energy stored in the body is equal to the work done by the applied load in stretching the body.

DEFINITIONS : Resilience

Resilience: The total strain energy stored in a body is commonly known as Resilience. Whenever the straining force is removed from the strained body, the body is capable of doing work. Hence the Resilience is also defined as the capacity / ability of a strained body for doing work on the removal of the straining force.

Resilience is the positive ability of a system or ~~recov~~ a physical property of a material that can resume its shape after being stretched or deformed. Elasticity.

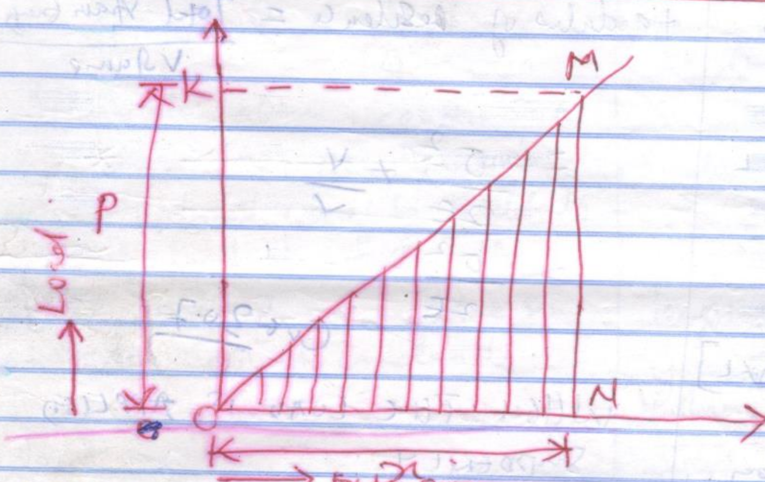
Proof Resilience: The maximum strain energy stored in a body, is known as proof resilience. The strain energy stored in the body will be maximum when the body is stressed upto elastic limit. Hence the proof resilience is the quantity of strain energy stored in a body when strained upto elastic limit

In physics – To cause distortion/deformation by applying an external force.

Modulus of Resilience: It is defined as the proof resilience of a material per unit volume. It is an important property of a material. Mathematically

$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

EXPRESSION FOR STRAIN ENERGY STORED IN A BODY WHEN THE LOAD IS APPLIED GRADUALLY:



The figure showed load extension diagram of a body under tensile ~~test~~ upto elastic limit. The tensile load P increases gradually from zero to the value of P and the extension of the body increases from 0 to x value.

The load P performs work in stretching the body. This work will be stored in the body as strain energy which is recoverable after the load P is removed.

let P = Gradually applied load
x = Extension of the body.
A = Cross-sectional area
L = length of the body
V = Volume of the body
E = Young's Modulus
σ = Stress induced in the body and

U = Strain energy stored in the body.

Now,
Work done by the load = Area of load extension curve [Shaded area]

i.e. = Area of Δ ONM

$= \frac{1}{2} \times P \times x$ ——(1)

But
load [force] P = Stress × Area

$P = \sigma A \left[\sigma = \frac{P}{A}\right]$

and extension, x = Strain × Length

from Strain $= \frac{x}{L}$

i.e

$$x = \frac{\text{Stress}}{E} \times L$$

from

$$\text{Strain} = \frac{\text{Stress}}{E}$$

i.e

$$x = \frac{\sigma}{E} \times L$$

Substituting the values of P and x in eqn (1)

i.e

$$\text{Work done} = \frac{1}{2} \times \sigma \cdot A \times \frac{\sigma L}{E}$$

$$= \frac{\sigma^2}{2E} A \cdot L$$

$$= \frac{\sigma^2}{2E} V \quad [V = A \times L \text{ volume}]$$

But the work done by the load in stretching the body is equal to strain energy stored in the body

∴ Energy stored in the body,

$$U = \frac{\sigma^2}{2E} \times V$$

Proof resilience : maximum energy stored in the body without permanent deformation (i.e upto elastic limit).

Hence if the stress σ is taken to the elastic limit, we get proof resilience

i.e Proof Resilience $= \frac{\sigma^{*2}}{2E} \times$ Volume

σ^* = stress upto elastic limit

$$\text{Modulus of Resilience} = \frac{\text{Total Strain Energy}}{\text{Volume}}$$

$$= \frac{\sigma^2}{2E} \times \frac{V}{V}$$

$$= \frac{\sigma^2}{2E}$$

WHEN THE LOAD IS APPLIED SUDDENLY

When the load is applied suddenly, the load (force) P is constant throughout the process of the deformation of the body when the extension of the body takes place.

∴

Work done by the load = Load × Extension

$= \cancel{P \times x}\ P x$

But we know that, the max ~~e~~ strain energy stored (i.e energy stored upto elastic limit) in a body is given by.

$$U = \frac{\sigma^2}{2E} \times V$$

7661

$$\frac{\sigma^2}{2E} \times A \times L = P\,x$$

$$\frac{\sigma^2}{2E} \times A \times L = P \times \frac{\sigma}{E} \times L$$

$$\frac{\sigma}{2} \times A = P$$

$$\sigma = 2 \times \frac{P}{A}$$

From the above equation it is clear that the maximum stress induced due to suddenly applied load is twice the stress induced when the same load is applied gradually.

Ex 1

A tensile load of 60kN is gradually applied to a circular bar of 4cm diameter and 5m long. If the value of $E = 2 \times 10^5 N/mm^2$ determine:

(i) Stretch in the rod

(ii) Stress in the rod

(iii) Strain energy absorbed by the rod

Solution

Gradually applied load $P = 60KN$
$= 60{,}000\,N$

Dia of rod $d = 4cm = 40mm$

$\therefore$ Area $= \frac{\pi D^2}{4}$

$A = \pi r^2 \quad r = \frac{D}{2}$

$A = \pi \left(\frac{D}{2}\right)^2 = \frac{\pi D^2}{4}$

$A = \frac{\pi}{4} \times 40^2$
$= 400\pi\ mm^2$

length of the rod $L = 5m = 5000mm$

Volume of the rod $V = A \times L$
$= 400\pi \times 5000$
$= 2 \times 10^6\ mm^3$

Young modulus $E = 2 \times 10^5 N/mm^2$

let x = stretch or extension in the rod

σ = stress in the rod

U = strain energy absorbed by the rod.

$\sigma = \frac{P}{A} = \frac{60000}{400\pi} = 47.746 N/mm^2$

$x = \frac{\sigma}{E} \times L$

$= \frac{47.746}{2 \times 10^6} \times 5000 = 1.19mm.$

$$U = \frac{\sigma^2}{2E} \times V$$

$$= \frac{47.746^2}{2 \times 2 \times 10^6} \times 2 \times 10^6 \pi$$

$$= 35810 \text{ N mm}$$

$$U = 35.81 \text{ N m}$$

(2) If in example 1, the tensile load of 60kN is applied suddenly determine:

(i) maximum instantaneous stress induced

(ii) instantaneous elongation in the rod

(iii) Strain energy absorbed in the rod.

(i) Max instantaneous stress induced

$$\sigma = 2 \times \frac{P}{A}$$

$$= 2 \times 47.746$$

$$= 95.493 \text{ N/mm}^2$$

(ii) Instantaneous elongation in the rod

$$x = \frac{\sigma}{E} \times L = \frac{95.493}{2 \times 10^6} \times 5000$$

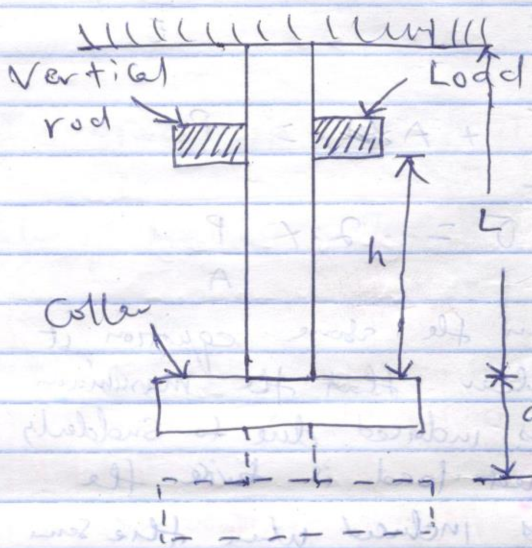
$$= 2.38 \text{ mm}$$

(iii) $U = \frac{\sigma^2}{2E} \times V = \frac{95.493^2 \times 2 \times 10^6 \pi}{2 \times 2 \times 10^6}$

143238 N mm = 143.238 N m.

CVE 307

EXPRESSION FOR STRAIN ENERGY STORED IN A BODY WHEN LOAD IS APPLIED WITH AN IMPACT



Consider a vertical rod fixed at the upper end and having a collar at the lower end as shown above.

Let the load be dropped from the height on the collar.

Due to this impact load, there will be an extension on the rod.

Let P = load dropped (i.e load applied with an impact.)

L = length of the rod

A = x-sectional area of the rod

V = Volume of the rod

h = Height ~~of~~ through which load is dropped

δl = Extension of the rod due to load P

E = Modulus of elasticity of the material rod

σ = Stress induced in the rod due to impact load.

Then the strain in the bar is given by

$$\text{Strain } \varepsilon = \frac{\text{Stress } \sigma}{E}$$

i.e. $\dfrac{\delta L}{L} = \dfrac{\sigma}{E}$

i.e. $\delta L = \dfrac{\sigma}{E} \times L$

Work done by the load = Load × distance moved

$= P(h + \delta L)$

The strain energy stored by the rod =

$$U = \frac{\sigma^2}{2E} \times V$$

i.e. $U = \dfrac{\sigma^2}{2E} \times AL$

~~Equating~~

But work done by the load = Strain energy stored by the rod.

i.e. $P(h + \delta L) = \dfrac{\sigma^2}{2E} \cdot AL$

$$P\left[h + \frac{\sigma}{E} \cdot L\right] = \frac{\sigma^2}{2E} \cdot AL$$

i.e. $Ph + P\dfrac{\sigma}{E} \cdot L = \dfrac{\sigma^2}{2E} \cdot AL$

i.e.

$$\frac{\sigma^2}{2E} \cdot AL - P \cdot \frac{\sigma}{E} \cdot L - Ph = 0$$

i.e. multiplying by $\dfrac{2E}{AL}$

i.e. $\sigma^2 - P \cdot \dfrac{\sigma}{E} \cdot L \cdot \dfrac{2E}{AL} - Ph \cdot \dfrac{2E}{AL} = 0$

i.e.

$$\sigma^2 - \frac{2P}{A}\sigma - \frac{2PEh}{AL} = 0$$

Solving Quadratically

$$\sigma = \frac{\frac{2P}{A} \pm \sqrt{\left(\frac{2P}{A}\right)^2 + 4 \cdot \frac{2PEh}{A \cdot L}}}{2 \times 1}$$

$$\cancel{\sigma_1} = \cancel{\frac{P}{A}} \pm \cancel{\sqrt{\frac{4P^2}{A^2} + 8\,PEh}}$$

$$\sigma = \frac{\frac{2P}{A} \pm \sqrt{\frac{4P^2}{A^2} + 4 \cdot \frac{2PEh}{A \cdot L}}}{2}$$

1.0

$$\sigma = \frac{\frac{2P}{A} \pm \left(2\sqrt{\frac{P^2}{A^2}} + 2\sqrt{\frac{2PEh}{AL}}\right)}{2}$$

$$\sigma = \frac{P}{A} \pm \sqrt{\left(\frac{P}{A}\right)^2 + \frac{2PEh}{A \cdot L}}$$

$$\cancel{= \frac{P}{A} + \frac{P}{A}\sqrt{1 + 2AEh}}$$

$$\sigma = \frac{P}{A} + \sqrt{\frac{P^2}{A^2} \times \frac{A^2}{P^2} + \frac{2PEh}{AL} \times \frac{A^2}{P^2}}$$

$$\cancel{= \frac{P}{A} + \sqrt{1 + \frac{2AEh}{PL}}}$$

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{P \cdot L}}\right)$$

After knowing value of σ the strain energy can be obtained

~~$\sigma = \frac{P}{A} + \frac{P}{A}$~~

neglecting -ve root

But

If δL is very small in comparison with h

Then

work done = $P \cdot h$

1.0

$$Ph = \frac{\sigma^2}{2E} AL$$

$$\sigma^2 = \frac{2EP \cdot h}{AL}$$

$$\sigma = \sqrt{\frac{2EPh}{A \cdot L}}$$

But in case $h = 0$

no height

we get

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1+0}\right)$$

$$\sigma = \frac{P}{A}(1+1)$$

$$\sigma = \frac{2P}{A}$$